T-79.230 **Kevät 2005**
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Solutions

1. The CURRENT-BEST-LEARNING algorithm produces the following results. FP denotes false positive and FN denotes false negative.

   (a) $\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{Hungry}(x))$
   $x_1 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Hungry}(x)$
   $x_2 : FP : \forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Hungry}(x) \wedge \mathit{Est}(x, 0-10)$
   $x_3 : FN : \forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10)$
   $x_4 : FN : \forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \vee \mathit{Est}(x, 10-30)$
   $x_5 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \vee \mathit{Est}(x, 10-30)$
   $x_6 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \vee \mathit{Est}(x, 10-30)$
   $x_7 : FP : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, 0-10) \wedge \mathit{Patrons}(\mathit{some})) \vee \mathit{Est}(x, 10-30)$
   $x_8 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, 0-10) \wedge \mathit{Patrons}(\mathit{some})) \vee \mathit{Est}(x, 10-30)$
   $x_9 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, 0-10) \wedge \mathit{Patrons}(\mathit{some})) \vee \mathit{Est}(x, 10-30)$
   $x_{10} : FP : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, 0-10) \wedge \mathit{Patrons}(\mathit{some})) \vee (\mathit{Est}(x, 10-30) \wedge \neg \mathit{Price}(x, \$\$\$))$
   $x_{11} : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, 0-10) \wedge \mathit{Patrons}(\mathit{some})) \vee (\mathit{Est}(x, 10-30) \wedge \neg \mathit{Price}(x, \$\$\$))$
   $x_{12} : FN : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, 0-10) \wedge \mathit{Patrons}(\mathit{some})) \vee (\mathit{Est}(x, 10-30) \wedge \neg \mathit{Price}(x, \$\$\$)) \vee (\mathit{Est}(x, 30-60) \wedge \mathit{Type}(x, \mathit{Burger}))$

   (b) $\forall x(\mathit{WillWait}(x) \leftrightarrow \neg\, \mathit{WaitEstimate}(x, \mathit{30–60}))$
   $x_1 : FN : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Est}(x, 30-60) \vee \mathit{Pat}(x, \mathrm{some}))$
   $x_2 : FP : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}))$
   $x_3 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \mathrm{some})$
   $x_4 : FN : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee \mathit{Fri}(x))$
   $x_5 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee \mathit{Fri}(x))$
   $x_6 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee \mathit{Fri}(x))$
   $x_7 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee \mathit{Fri}(x))$
   $x_8 : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee \mathit{Fri}(x))$
   $x_9 : FP : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$
   $x_{10} : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$
   $x_{11} : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$
   $x_{12} : -- : \forall x\, \mathit{WillWait}(x) \leftrightarrow (\mathit{Pat}(x, \mathrm{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$

2. Surprise candy comes in two flavors, cherry and lime. 4 pieces have been opened, out of which 3 were cherry.

a 100% cherry

b 75% cherry and 25% lime

c 50% cherry and 50% lime

d 25% cherry and 75% lime

e 100% lime

(a) Which is the most likely (ML) hypothesis?

Let $\mathbf{d} =$ "3 cherries and 1 lime".

$P(\mathbf{d} \mid h_a) = 0$
$P(\mathbf{d} \mid h_b) = 4 \cdot (\frac{3}{4})^3 \cdot \frac{1}{4} = \frac{27}{64} \approx 0,42$
$P(\mathbf{d} \mid h_c) = 4 \cdot (\frac{1}{2})^3 \cdot \frac{1}{2} = 0,25$
$P(\mathbf{d} \mid h_d) = 4 \cdot (\frac{1}{4})^3 \cdot \frac{3}{4} = \frac{3}{64} \approx 0,047$
$P(\mathbf{d} \mid h_e) = 0$

Thus b is the most likely hypothesis.

(b) Suppose that the prior distribution of the bags is $\langle 0.1, 0.1, 0.1, 0.6, 0.1 \rangle$.
Find out the maximum a posteriori (MAP) hypothesis

$P(h_a) = 0,1$
$P(h_b) = 0,1$
$P(h_c) = 0,1$
$P(h_d) = 0,6$
$P(h_e) = 0,1$

$P(h_a \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_a) P(h_a) = 0$
$P(h_b \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_b) P(h_b) = 0.042$
$P(h_c \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_c) P(h_c) = 0.025$
$P(h_d \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_d) P(h_d) = 0.028$
$P(h_e \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_e) P(h_e) = 0$

$P(h_a \mid \mathbf{d}) = 0$
$P(h_b \mid \mathbf{d}) = \frac{0,042}{0,042+0,025+0,028} = 0,44$
$P(h_c \mid \mathbf{d}) = \frac{0,025}{0,042+0,025+0,028} = 0,26$
$P(h_d \mid \mathbf{d}) = \frac{0,028}{0,042+0,025+0,028} = 0,29$
$P(h_e \mid \mathbf{d}) = 0$

The maximum a posteriori hypothesis is h_b.

(c) Estimate the probability that the fifth piece of candy is lime flavored

$P(\text{lime}_5 \mid \mathbf{d}) = \frac{1}{4} P(h_b \mid \mathbf{d}) + \frac{1}{2} P(h_c \mid \mathbf{d}) + \frac{3}{4} P(h_d \mid \mathbf{d})$

Using the prior distribution of bags in part (b) gives us the probability of getting a lime from the bag in fifth take as:

$P(\text{lime}_5 \mid \mathbf{d}) = 0.25 \cdot 0,44 + 0.5 \cdot 0,26 + 0.75 \cdot 0,29 = 0,46$