T-79.230 **Kevät 2004**
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Solutions

1. The CURRENT-BEST-LEARNING algorithm produces the following results. FP denotes false positive and FN denotes false negative.

   (a) $\forall x(\mathit{WillWait}(x) \leftrightarrow Hungry(x))$
   x_1 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow Hungry(x)$
   x_2 : FP : $\forall x\, \mathit{WillWait}(x) \leftrightarrow Hungry(x) \wedge \mathit{Est}(x, 0-10)$
   x_3 : FN : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10)$
   x_4 : FN : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \vee \mathit{Est}(x, 10-30)$
   x_5 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \vee \mathit{Est}(x, 10-30)$
   x_6 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \vee \mathit{Est}(x, 10-30)$
   x_7 : FP : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \wedge \neg \mathit{Burger}) \vee \mathit{Est}(x, 10-30)$
   x_8 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \wedge \neg \mathit{Burger}) \vee \mathit{Est}(x, 10-30)$
   x_9 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \wedge \neg \mathit{Burger}) \vee \mathit{Est}(x, 10-30)$
   x_{10} : FP : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \wedge \neg \mathit{Burger}) \vee (\mathit{Est}(x, 10-30) \wedge \neg \mathit{Price}(x, \$\$\$))$
   x_{11} : FP : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \wedge \neg \mathit{Burger} \wedge \neg \mathit{Patrons}(x, \text{none})) \vee (\mathit{Est}(x, 10-30) \wedge \neg \mathit{Price}(x, \$\$\$))$
   x_{12} : FN : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 0-10) \wedge \neg \mathit{Burger} \wedge \neg \mathit{Patrons}(x, \text{none})) \vee (\mathit{Est}(x, 10-30) \wedge \neg \mathit{Price}(x, \$\$\$)) \vee (\mathit{Bar}(x) \wedge Hungry(x))$

   (b) $\forall x(\mathit{WillWait}(x) \leftrightarrow \mathit{WaitEstimate}(x, \mathit{30–60}))$
   x_1 : FN : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Est}(x, 30-60) \vee \mathit{Pat}(x, \text{some}))$
   x_2 : FP : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}))$
   x_3 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some})$
   x_4 : FN : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Fri}(x))$
   x_5 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Fri}(x))$
   x_6 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Fri}(x))$
   x_7 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Fri}(x))$
   x_8 : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee \mathit{Fri}(x))$
   x_9 : FP : $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$
   x_{10} : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$
   x_{11} : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$
   x_{12} : $--$: $\forall x\, \mathit{WillWait}(x) \leftrightarrow \mathit{Pat}(x, \text{some}) \vee (\mathit{Fri}(x) \wedge \neg \mathit{Est}(x, > 60))$

2. Surprise candy comes in two flavors, cherry and lime. 4 pieces have been opened, out of which 3 were cherry.

   a 100% cherry

   b 75% cherry and 25% lime

c 50% cherry and 50% lime

d 25% cherry and 75% lime

e 100% lime

(a) Which is the most likely (ML) hypothesis?
Let $X =$ "3 cherries and 1 lime".
$P(X|a) = 0$
$P(X|b) = 4 \cdot (\frac{3}{4})^3 \cdot \frac{1}{4} = \frac{27}{64} \approx 0,42$
$P(X \mid c) = 4 \cdot (\frac{1}{2})^3 \cdot \frac{1}{2} = 0,25$ $P(X|d) = 4 \cdot (\frac{1}{4})^3 \cdot \frac{3}{4} = \frac{3}{64} \approx 0,047$
$P(X|e) = 0$

Thus b is the most likely hypothesis.

(b) Suppose that the prior distribution of the bags is $\langle 0.1, 0.1, 0.1, 0.6, 0.1 \rangle$.
Find out the maximum a posteriori (MAP) hypothesis
$P(h_a) = 0,1$
$P(h_b) = 0,1$
$P(h_c) = 0,1$
$P(h_d) = 0,6$
$P(h_e) = 0,1$

$\alpha P(X|h_a) = P(h_a|X) \cdot P(h_a) = 0$
$\alpha P(X|h_b) = P(h_b|X) \cdot P(h_b) = 0,042$
$\alpha P(X|h_c) = P(h_c|X) \cdot P(h_c) = 0,025$
$\alpha P(X|h_d) = P(h_d|X) \cdot P(h_d) = 0,028$
$\alpha P(X|h_e) = P(h_e|X) \cdot P(h_e) = 0$
$P(X|h_a) = 0$
$P(X|h_b) = \frac{0,042}{0,042+0,025+0,028} = 0,44$
$P(X|h_c) = \frac{0,025}{0,042+0,025+0,028} = 0,26$
$P(X|h_d) = \frac{0,028}{0,042+0,025+0,028} = 0,29$
$P(X|h_e) = 0$
The maximum a posteriori hypothesis is b.

(c) Estimate the probability that the fifth piece of candy is lime flavored
$P(\text{lime}_5|\text{history}) = \frac{1}{4}P(X|h_b) + \frac{1}{2}P(X|h_c) + \frac{3}{4}P(X|h_d)$
Using the prior distribution of bags in part (b) gives us the probability of getting a lime from the bag in fifth take as:
$= \frac{1}{4} \cdot 0,44 + \frac{1}{2} \cdot 0,26 + \frac{3}{4} \cdot 0,29 = 0,46$