**T-79.230** **Kevät 2004**
**Agenttipohjaisen tietojenkäsittelyn perusteet**
**Laskuharjoitus 4**
**Solutions**

1. a) The costs of the tree routes are as follows:

| Route | Time (min) | Cost (mk) |
|---|---|---|
| $I$ | 57 | 39 |
| $II$ | 33 | 26 |
| $III$ | 55 | 20 |

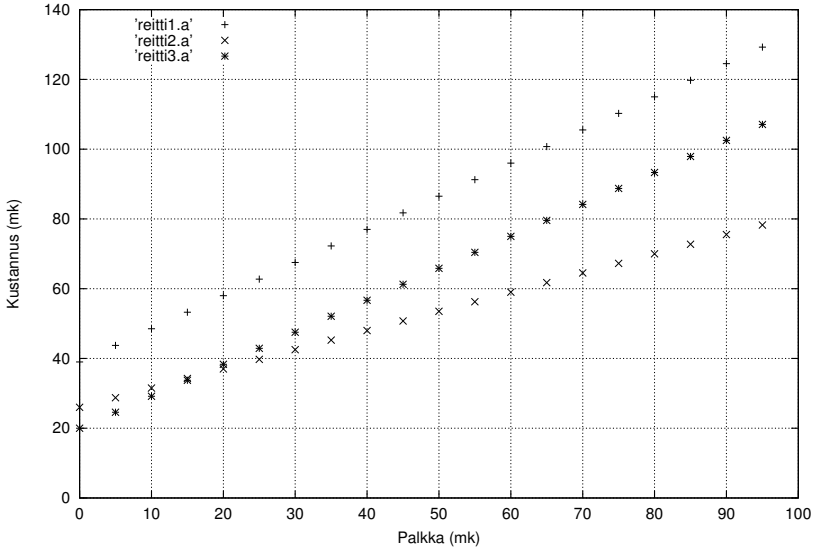
If the engineer salary $a = 40$ mk, the cost function $U(t, m) = m + at$ gives us the following values:

$$I : U(57, 39) = 39 + \frac{57}{60} \cdot 40 = 77$$
$$II : U(33, 26) = 26 + \frac{33}{60} \cdot 40 = 48$$
$$III : U(55, 20) = 20 + \frac{55}{60} \cdot 40 = 56.7$$

With given parameters the route $II$ is the best.
We find the point where $III$ is better than $II$ by solving the following inequality:

$$\frac{33}{60}x + 26 \geq \frac{55}{60}x + 20$$
$$x \leq 16.36,$$

so the engineer had to earn less than 16.36 mk/h for route $III$ to be better.
The figure below gives the costs of different routes when $a$ varies between 0–100.



We see that $I$ is dominated by the two other routes so it can be left out of decision process.

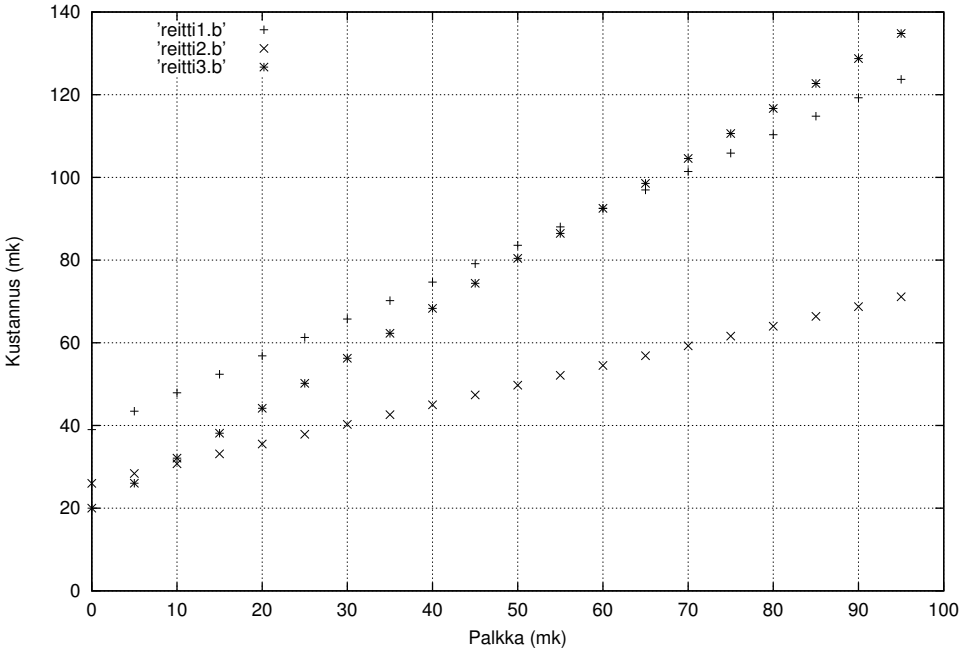
b) When we use the cost function $U(t_1, t_2, m) = a_1 t_1 + a_2 t_2 + m$, where $a_1 = 1.5a$ and $a_2 = 0.5a$ the situation is as follows:

| Route | Time $t_1$ (min) | Time $t_2$ (min) | Cost (mk) |
|---|---|---|---|
| $I$ | 25 | 32 | 39 |
| $II$ | 12 | 21 | 26 |
| $III$ | 45 | 10 | 20 |

Thus, the costs of the routes are:

$$I : U(25, 32, 39) = 39 + \frac{25}{60} \cdot 60 + \frac{32}{60} \cdot 20 = 74.7$$
$$II : U(12, 21, 26) = 26 + \frac{12}{60} \cdot 60 + \frac{21}{60} \cdot 20 = 45$$
$$III : U(45, 10, 20) = 20 + \frac{45}{60} \cdot 60 + \frac{10}{60} \cdot 20 = 74.375$$

Again, $II$ was better than the others. The following figure shows how the costs change as function of salary:




In this interval none of the options dominates the other ones.

c) If the results of choices are not deterministic, we use the expected value $E[U(X)]$ of the utility function as basis for the decisions.

The probability distribution of the three different routes is:

| Route | $t$ (min) | $p(t)$ |
|---|---|---|
| $I$ | 57 | 0.75 |
| | 58 | 0.20 |
| | 62 | 0.05 |
| $II$ | 33 | 0.30 |
| | 34 | 0.20 |
| | 43 | 0.20 |
| | 48 | 0.30 |
| $III$ | 55 | 0.16 |
| | 56 | 0.19 |
| | 57 | 0.03 |
| | 60 | 0.17 |
| | 61 | 0.04 |
| | 65 | 0.17 |
| | 66 | 0.03 |
| | 70 | 0.17 |
| | 71 | 0.03 |
| | 75 | 0.01 |

This gives us the following expected values and costs:

| Reitti | $E(t)$ (min) | $U(t,m)$ (mk) |
|---|---|---|
| $I$ | 57.45 | 77.3 |
| $II$ | 39.7 | 52.47 |
| $III$ | 61.6 | 61.06 |

Again, we should choose route $II$.