T-79.230 **Kevät 2003**
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Laskuharjoitus 3
Solutions

1. a) The costs of the tree routes are as follows:

Route	Time (min)	Cost (mk)
I	57	39
II	33	26
III	55	20

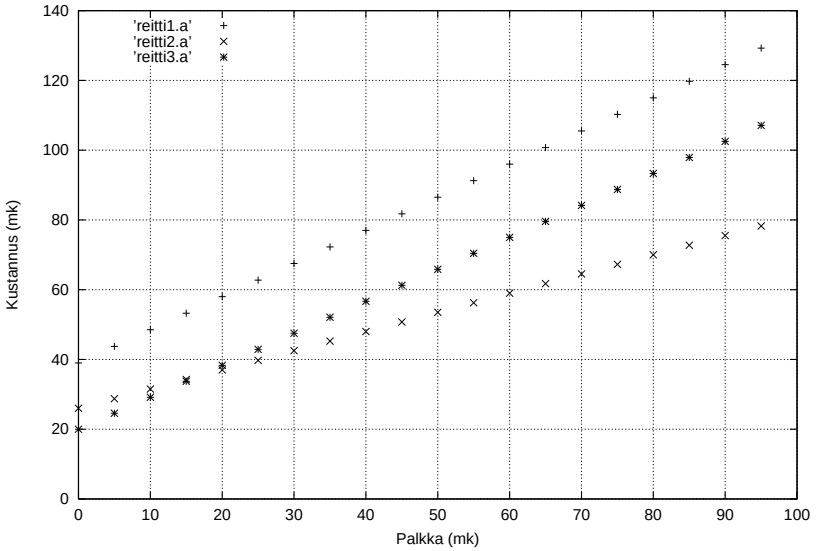
If the engineer salary $a = 40$ mk, the cost function $U(t, m) = m + at$ gives us the following values:

$$I : U(57, 39) = 39 + \frac{57}{60} \cdot 40 = 77$$
$$II : U(33, 26) = 26 + \frac{33}{60} \cdot 40 = 48$$
$$III : U(55, 20) = 20 + \frac{55}{60} \cdot 40 = 56.7$$

With given parameters the route II is the best.
We find the point where III is better than II by solving the following inequality:

$$\frac{33}{60}x + 26 \geq \frac{55}{60}x + 20$$
$$x \leq 16.36,$$

so the engineer had to earn less than 16.36 mk/h for route III to be better.
The figure below gives the costs of different routes when a varies between 0–100.



We see that I is dominated by the two other routes so it can be left out of decision process.

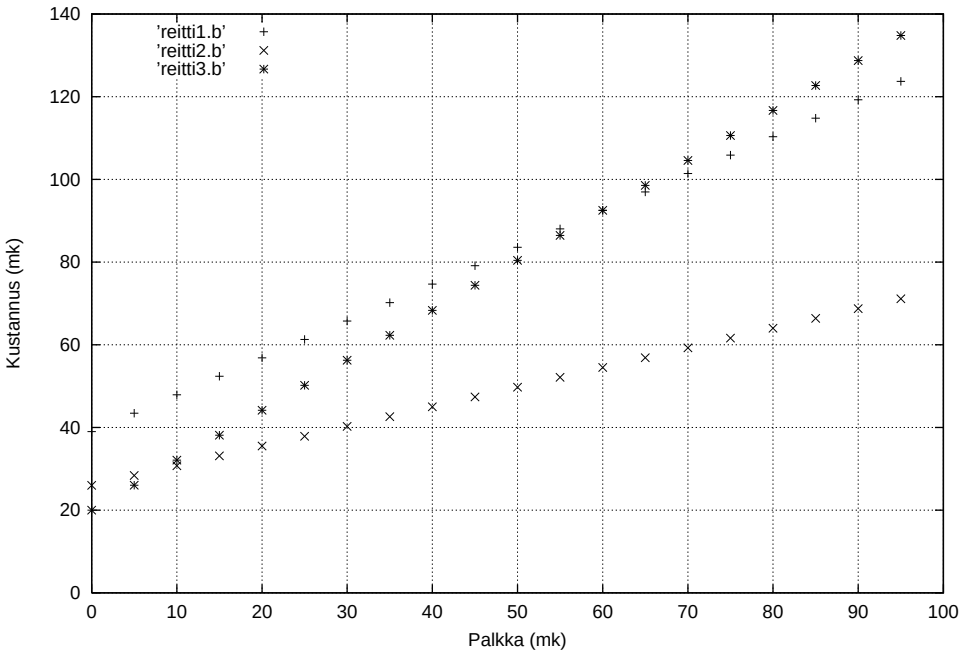
b) When we use the cost function $U(t_1, t_2, m) = a_1 t_1 + a_2 t_2 + m$, where $a_1 = 1.5a$ and $a_2 = 0.5a$ the situation is as follows:

Route	Time t_1 (min)	Time t_2 (min)	Cost (mk)
I	25	32	39
II	12	21	26
III	45	10	20

Thus, the costs of the routes are:

$$I : U(25, 32, 39) = 39 + \frac{25}{60} \cdot 60 + \frac{32}{60} \cdot 20 = 74.7$$
$$II : U(12, 21, 26) = 26 + \frac{12}{60} \cdot 60 + \frac{21}{60} \cdot 20 = 45$$
$$III : U(45, 10, 20) = 20 + \frac{45}{60} \cdot 60 + \frac{10}{60} \cdot 20 = 74.375$$

Again, II was better than the others. The following figure shows how the costs change as function of salary:

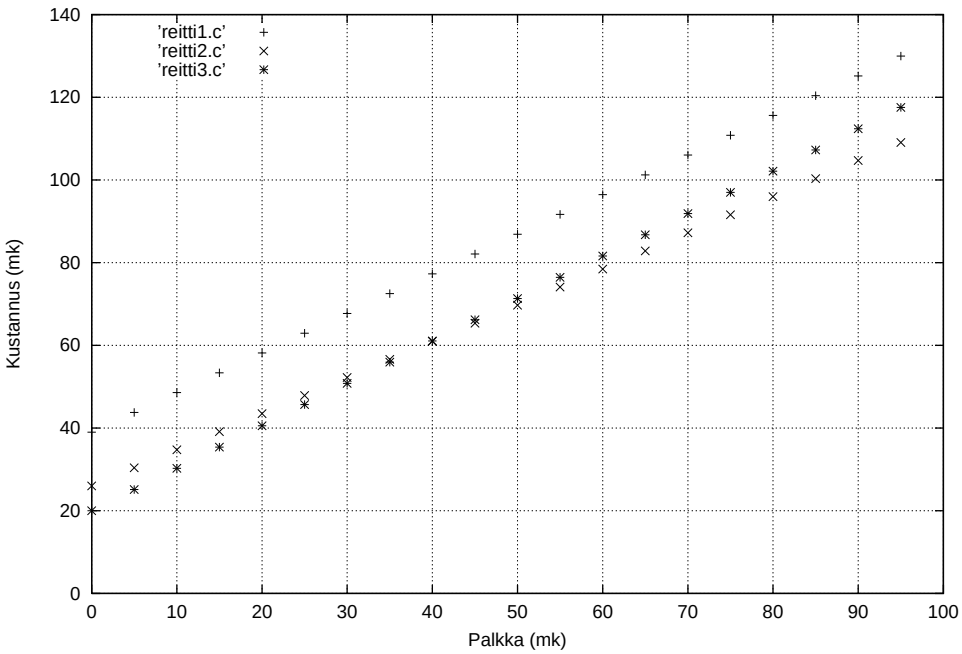


In this interval none of the options dominates the other ones.

c) If the results of choices are not deterministic, we use the expected value $E[U(X)]$ of the utility function as basis for the decisions.
The probability distribution of the three different routes is:

Route	t (min)	$p(t)$
I	57	0.75
	58	0.20
	62	0.05
II	33	0.30
	34	0.20
	43	0.20
	48	0.30
III	55	0.16
	56	0.19
	57	0.03
	60	0.17
	61	0.04
	65	0.17
	66	0.03
	70	0.17
	71	0.03
	75	0.01

This gives us the following expected values and costs:

Reitti	$E(t)$ (min)	$U(t,m)$ (mk)
I	57.45	77.3
II	39.7	52.47
III	61.6	61.06

Again, we should choose route II.