Summary on Improving Wireless Sensor Network Lifetime through Power Aware Organization by Mihaela Cardei and Ding-Zhu Du

Lasse Kiviluoto

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1 Introduction

The lifetime of wireless sensor network is limited due to use of some kind of battery. To be able to reduce the energy consumption and so improve the lifetime of wireless sensor network many different kind of methods has been developed in last few years. These energy efficient can be found in all the layers of the protocol stack. Mostly these methods methods can be divided in following categories:

- schedule operations, to allow the nodes to enter low energy sleep states
- choose routes that consumes the lowest energy
- selectively use wireless nodes based on their energy status
- reduce the amount of data and avoid useless activity

In the paper [1] Cardei and Du introduce a new energy saving method were the sensor nodes are divided into disjoint sets so that every set covers every target we want to monitor. Now lot of energy can be spared, if we activate only one of these sets and put the other sensors to a low-energy sleep state.

To be able to save energy using the disjoint sets, we have to make some assumptions. It is assumed that a large number of sensors are dispersed randomly around the objectives we want to monitor. We want to monitor every target all the time by at least one sensor. Every sensor is able to monitor all the targets within its operational range. It is assumed that the target locations are fixed and so the algorithm that computes the sets has to be executed only once by a central node. When the sensors are deployed they activate their positioning service and send their location information to a central node. The central node calculates the disjoint sets and sends membership information to every sensor. Every sensor is able to determine the sleep time periods using the membership information and the number of covers. Because this system depends on time periods, we have to assume that a time synchronization service is available. This service can arranged by a periodic beacon message from the central node or by an on-board GPS receiver.

2 Disjoint Set Covers Problem

In disjoint set covers problem we assume that n sensors $S_1, S_2, ..., S_n$ are deployed in territory to monitor m targets $T_1, T_2, ..., T_m$. Our goal is to divide these sensors into a maximum number of disjoint sets so that every set completely covers all the targets. Target is considered to be covered when there is a active sensor that has the target in its sensing range.

Disjoint set covers problem can be modelled as a collection of sensors $C = \{S_1, S_2, ..., S_n\}$, where every sensor covers a subset of the targets $T = \{T_1, T_2, ..., T_n\}$. So we can write every sensor S_i to be a subset $S_i = \{T_{i_1}, T_{i_2}, ..., T_{i_l}\}$ of targets. Now the disjoint set covers problem can be defined as follows.

Definition 1 (Disjoint Set Covers Problem (DSC) [1]). *Given a collection C of subsets of a finite set T, find the maximum number of disjoint covers for T. Every cover C_i is a subset of C, $C_i \subseteq C$, such that every element of T belongs to at least one member of C_i, and for any two covers C_i and C_j, $C_i \cap C_j = \emptyset$.*

This problem can be seen as a generalization of the minimum cover problem. The minimum cover problem is NP-complete and so is this generalized version, DSC.

Theorem 1. *DSC is NP-complete*

Proof. See [1]. □

3 Heuristic for maximum disjoint set cover problem

To compute the maximum number of covers, the DSC problem is transformed into a maximum-flow problem. This maximum-flow problem (MFP) is solved using a mixed integer programming (MIP). Based on this MIP specific heuristic is used to compute the disjoint covers.

The transformation from DSC problem into a MFP is done in following four steps.

- Consider a bipartite directed graph $G = (V, E)$, where the vvertex set $V = C \cup T$ and $S_iT_j \in E$ iff T_j is in S_i, where $1 \leq i \leq n$ and $1 \leq j \leq m$. Then assign to every edge S_iT_j a capacity $c_{S_iT_j} = 1$, Create a vertex X and conenct every vertex T_j in T to X with an edge of capacity 1.

- Find a critical element in T which is contained by a minimum number of subsets in the collection C and and note this number with k. Draw k copies of G, namely $G_1, G_2, ..., G_k$. In these k copies (components), let the first index in a vertex notation reflect the component it belongs to e.g. a vertex S_i in G, is named $S_{1i}, S_{2i}, ..., S_{ki}$ in $G_1, G_2, ..., G_k$.

- Create a source node S and for each S_i in C, create a vertex S_{0i}. Then connect the source S with S_{0i} with an edge with capacity equal with the degree of S_i in G. Also, connect S_{0i} with S_{ji} for any $1 \leq j \leq k$ and assign a capacity equal with the degree of S_i in G.

- Create two sinks Y_1 and Y_2. Connect each vertex X_j with $1 \leq j \leq k$ to Y_2 and assign a capacity m. Then connect every vertex T_{ij} with $1 \leq i \leq k$ and $1 \leq j \leq m$ to Y_1 and assign the capacity n.

The flow f is an integer-valued function, that satisfies two properties. First of all the flow constrain has to be satisfied, every edge $uv \in E$, $0 \leq f_{uv} \leq c_{uv}$. Also in addition to a normal flow network we want that for every vertex $v \neq Y_1$, $f_{uv} \in \{0, c_{uv}$. The second property is the flow conservation, every vertex $u \in V - \{S, Y_1, Y_2\}$, $\sum_{v \in V, uv \in E \text{ or } vu \in E} f_{uv} = 0$. Now the problem is to maximize the flow received in vertex Y_2.

In this flow graph the copies of the bipartite digraph G represent different possible disjoint covers. There are k copies, because the k is clearly the upper-bound for the number of disjoint covers. The vertices $S_{01}, S_{02}, ..., S_{0n}$ ensure that the every sensor belongs to at most in one of these covers, because the flow $f_{uv} \in \{0, c_{uv}\}$ for every vertex $v \neq Y_1$. If the flow $f_{X_iY_2} = c_{X_iY_2}$, it is ensured that the flow in this copy of the digraph G really covers all the targets. The sink Y_1 is used to collect the flow generated by T_{ij} when the whole target set is not covered or some targets are covered by more than one sensor.

Theorem 2 ([1]). *Given a collection $C = \{S_1, S_2, ..., S_n\}$ of subsets of a finite set $T = \{T_1, T_2, ..., T_n\}$, the DSC problem return c^* covers if and only if the maximum-flow problem obtains the flow $c * m$ in Y_2.*

Proof. See [1]. □

This maximum-flow problem can be solved using mixed integer programming, which can be formulated as follows.

$$\begin{array}{lll}
\text{maximize} & f_{Y_2} & \\
\text{subject to} & (1)\ f_{uv} \leq c_{uv} & uv \in E \\
& (2)\ \sum_{u:uv\in E} f_{uv} - \sum_{u:vu\in E} f_{vu} = 0 & v \in V; v \neq \{S, Y_1, Y_2\} \\
& (3)\ f_{S_{pi}T_{pi_1}} = ... = f_{S_{pi}T_{pi_j}} & i = 1, ..., n; p = 1, ..., k; \\
& & S_i = \{T_{i_1}, ..., T_{i_j}\}; i_j = |S_i| \\
& (4)\ f_{T_{p1}X_p} = ... = f_{T_{pm}X_p} & p = 1, ..., k \\
& (5)\ f_{uv} \geq 0 & uv \in E
\end{array}$$

such that

- $f_{S_{pi}T_{pr}} \in \mathbb{N}$, for any $i = 1, ..., n, p = 1, ..., k$ and r such that $T_r \in S_i$
- $f_{T_{pj}X_p} \in \mathbb{N}$, for any $p = 1, ..., k$ and $j = 1, ..., m$
- all other flow variables $\in \mathbb{R}$.

In this problem the flow $f_{Y_2} = \sum_{i=1,...,k} f_{X_iY_2}$. Rows (1) and (5) assure the flow constrain and the row (2) is the flow conservation property of the flow network. Finally rows (3) and (4) assure that for every vertex $v \neq Y_1$ the flow $f_{uv} \in \{0, c_{uv}\}$. From the result of this mixed integer program we can construct the disjoint covers using the algorithm 1.

4 Evaluation

To test the performance of the MC-MIP to tests were made by Cardei and Du. This was done by simulating a stationary network with sensors and target points that were randomly located in a $500m$ x $500m$ area. It was assumed that the transmission range is equal for every sensor in the network.

The MC-MIP heuristic was compared to a heuristic called most constrained-minimally constraining heuristic. This latter heuristic uses the definition of fields (see [2]). We can simply compare these two heuristic by simply viewing

Algorithm 1: MC-MIP Heuristic

compute f_{Y_2} using MIP
$\alpha = f_{Y_2}/m; h = 0$
for *each* $p = 1, ..., k$ **do**
    if $f_{X_p Y_2} \neq 0$ **then**
        $h++; C_h = \emptyset$
        for *each* $i = 1, ..., n$ **do**
            if $f_{S_{0i} S_{pi}} \neq 0$ **then**
                $C_h = C_h \cup S_i$
            endif
        endfor
    endif
endfor
return *the disjoint covers* $C_1, C_2, ..., C_\alpha$

every field as a target. For more information about the most constrained-minimally constraining heuristic reader is referred to [2].

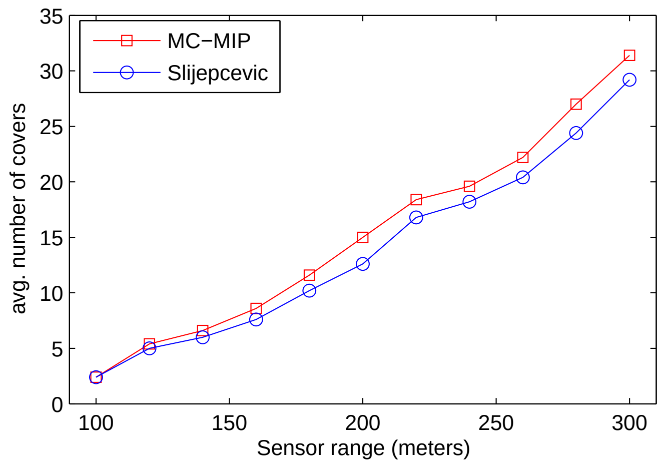

Figure 1: Average number of covers with 90 sensors and 10 targets

In the first experiment 10 randomly distributed targets were distributed. These targets were monitored using 50-90 sensors and having the sensing range $100m$-$300m$. For every parameter combination the experiments were repeated 5 times. In the second experiment there were 10-50 target points and 50-90 sensors with $250m$ sensing range. Again these experiments were repeated 5 times. In figure 1 we see the results the first experiment comparing the two heuristics. In figure 2 we have results from the second experiment. Clearly it can be seen that the MC-MIP has larger average number of covers, but in the tests the Slijepcevic's most constrained-minimally constraining heuristic was faster to compute.

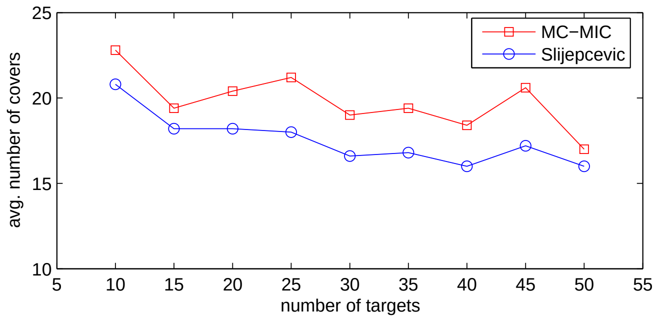

Figure 2: Average number of covers with 90 sensors sensing range of $250m$

5 Conclusions and remarks

In the paper [1] present new way to organize the sensors into disjoint set covers. If we activate only one of the covers at a time and leave the rest of sensors sleep state, energy savings can be archived and the lifetime of the network will increase. They have showed a heuristic to calculate the maximum disjoint set covers. The results show this method is clearly efficient method to organize the sensors.

All though the dividing the sensors into the disjoint set is a good idea, I have to remark that if we have only one of the covers activate it could be possible that there is no route to the central node anymore, because most of the sensors are in sleep state and their radio transceiver is turned off-line.

In addition because the DSC problem is NP-complete solving the problem using mixed integer programming is slow. It would be nice to see how large problems we can solve and what kind of networks are the hard ones to calculate. Also it has to be noted that if one of the sensors fail, the solution might not be optimal anymore and more likely the sensors remaining in that set might not cover all the targets. So the efficiency of this heuristic might not be so good in real life solutions.

References

[1] Mihaela Cardei, Ding-Zhu Du, Improving Wireless Sensor Network Lifetime through Power Aware Organization, *ACM Wireless Networks*, Vol. 11, 2005

[2] S. Slojepcevic, M. Potkonjak, Power Efficient Organization of Wireless sensor Networks, *IEEE International Conference on Communications*, June 2001, Helsinki, Finland