**T-79.194 Seminar on Theoretical Computer Science, Spring 2004**
**Solutions to Home Assignments (Emilia Oikarinen)**

1. (a) (max 0.5 points) Constraint graph $G$ is in Figure 1 (solid lines). The nodes correspond to variables of $\mathcal{P}$ and edges correspond to constraints of $\mathcal{P}$ (not explicitely labeled in $G$).

   (b) (max 0.5 points) One possible triangulation of $G$ is in Figure 1 (solid and dashed lines).

   (c) (max 1.0 points) One possible tree-decomposition of a triangulation of $G$ is in Figure 2.

2. (a) (max 1.0 points) $X$ is not a compatible enumeration order, since $d \preceq e$, $d \in \mathcal{C}_3$ and $e \in \mathcal{C}_2$. Thus condition "$\forall x \in \mathcal{C}_i, \forall y \in \mathcal{C}_j, i < j$ it holds $x \preceq y$" does not hold.

   For example $a \preceq b \preceq e \preceq c \preceq d$ is a compatible enumeration order for $X$.

   (b) (max 1.0 points) [Jégon and Ternioux, 2003]. Let $\mathcal{A}$ be a consistent instantiation of $Y$ such that $\mathcal{A}[\mathcal{C}_i \cap \mathcal{C}_j] = \mathrm{g}(\mathcal{C}_i/\mathcal{C}_j)$. By the definition of goods, there exists a consistent instantiation $\mathcal{B}$ on $\mathrm{Desc}(\mathcal{C}_j)$ such that $\mathcal{B}[\mathcal{C}_i \cap \mathcal{C}_j] = \mathrm{g}(\mathcal{C}_i/\mathcal{C}_j) = \mathcal{A}[\mathcal{C}_i \cap \mathcal{C}_j]$. Since $\mathcal{B}[\mathcal{C}_i \cap \mathcal{C}_j] = \mathcal{A}[\mathcal{C}_i \cap \mathcal{C}_j]$, by Lemma 1 [Jégon and Ternioux, 2003, p. 51], $\mathcal{A}$ and $\mathcal{B}$ are compatible. Therefore $\mathcal{B}$ is a consistent extension of $\mathcal{A}$ on $\mathrm{Desc}(\mathcal{C}_j)$. $\square$

[Jégon and Ternioux, 2003] P. Jégou and C. Terrioux, *Hybrid Backtracking Bounded by Tree-Decomposition of Constraint Networks.* Artificial Intelligence 146, pp. 43–75, 2003.

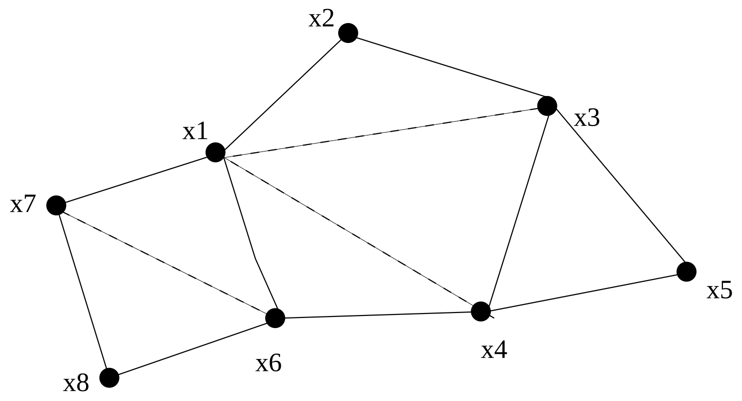


Figure 1: 1(a) and (b).

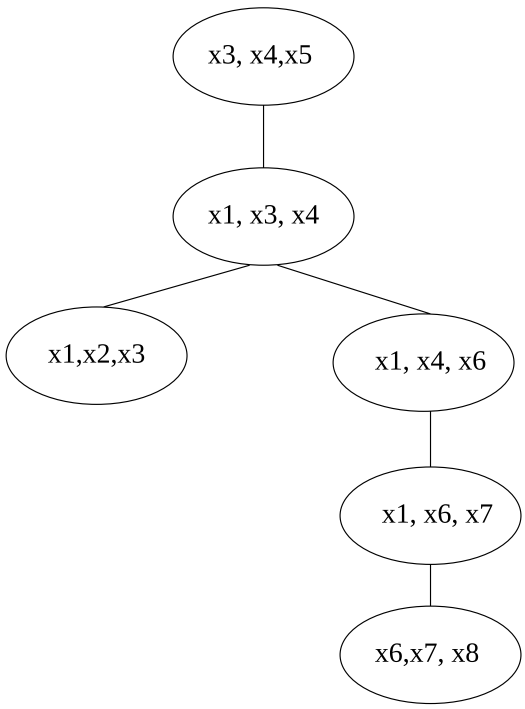


Figure 2: 1(c).