1. Consider the following set of linear inequalities:

\[-y \leq 0 \quad (1)\]
\[-y - z + 2 \leq x \quad (2)\]
\[0 \leq x \quad (3)\]
\[-y - 2 \leq x \quad (4)\]
\[y - 3 \leq x \quad (5)\]
\[x \leq -2y + 6 \quad (6)\]

Apply once the \textit{x-ELIMINATION} rule (to all possible inequalities in this set). Write down the resulting set of inequalities.
Answer

Using the $x$-ELIMINATION rule we obtain the following set of inequalities in which $x$ does not appear:

\begin{align*}
-y &\leq 0 \\
-y - z + 2 &\leq -2y + 6 \\
0 &\leq -2y + 6 \\
-y - 2 &\leq -2y + 6 \\
y - 3 &\leq -2y + 6
\end{align*}
2. Consider the following set of linear inequalities:

\[ 0 \leq y \quad (12) \]
\[ -x - y + 2 \leq z \quad (13) \]
\[ 0 \leq x \quad (14) \]
\[ -x - y \leq 2 \quad (15) \]
\[ -x + y \leq 3 \quad (16) \]
\[ x + 2y \leq 6 \quad (17) \]

Examine by using Fourier-Motzkin Elimination, whether this set is consistent or not. Write down the details.
Answer

Transforming each of them to the x-normal form yields the following set:

\[-y \leq 0 \quad (18)\]
\[-y - z + 2 \leq x \quad (19)\]
\[0 \leq x \quad (20)\]
\[-y - 2 \leq x \quad (21)\]
\[y - 3 \leq x \quad (22)\]
\[x \leq -2y + 6 \quad (23)\]
Using the \textit{x-ELIMINATION} rule we obtain the following set of inequalities in which \( x \) does not appear:

\begin{align*}
-y & \leq 0 \quad (24) \\
-y - z + 2 & \leq -2y + 6 \quad (25) \\
0 & \leq -2y + 6 \quad (26) \\
-y - 2 & \leq -2y + 6 \quad (27) \\
y - 3 & \leq -2y + 6 \quad (28)
\end{align*}
Transforming each of the five inequalities to the $y$-normal form we now obtain the following set:

$$
0 \leq y \quad (29) \\
y \leq z + 4 \quad (30) \\
y \leq 3 \quad (31) \\
y \leq 8 \quad (32) \\
y \leq 3 \quad (33)
$$

Eliminating now $y$ using the $y$-ELIMINATION rule we obtain the following set of four inequalities:

$$
0 \leq z + 4 \quad (34) \\
0 \leq 3 \quad (35) \\
0 \leq 8 \quad (36) \\
0 \leq 3 \quad (37)
$$
We can now delete the last three inequalities using the \textit{DELETION} rule and we end up with a single inequality the \( z \)-normal form which is:

\[ -4 \leq z \]

At this moment we apply the \textit{z-ELIMINATION} rule. We end up with the empty set. This implies that the original set of inequalities is consistent.