Some Incomplete Constraint Solvers
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1.
a)

$$\langle x_1 \neq x_3, x_1 = x_6, \underline{x_4 = x_4}, x_3 \neq x_6, x_5 \neq x_4, x_5 \neq x_7; x_1 \in \{a,b,c,d\}, x_2 = n, x_3 = d, x_4 \in \{e,r\}, x_5 = e, x_6 \in \{a,r,s\}, x_7 \in \{e,s\} \rangle$$

EQUALITY 1

$$\langle \underline{x_1 \neq x_3}, x_1 = x_6, x_3 \neq x_6, x_5 \neq x_4, x_5 \neq x_7; x_1 \in \{a,b,c,\underline{d}\}, x_2 = n, x_3 = d, x_4 \in \{e,r\}, x_5 = e, x_6 \in \{a,r,s\}, x_7 \in \{e,s\} \rangle$$

DISEQUALITY 3

$$\langle \underline{x_1 = x_6}, x_3 \neq x_6, x_5 \neq x_4, x_5 \neq x_7; x_1 \in \{a,\underline{b,c}\}, x_2 = n, x_3 = d, x_4 \in \{e,r\}, x_5 = e, x_6 \in \{a,\underline{r,s}\}, x_7 \in \{e,s\} \rangle$$

EQUALITY 2

$$\langle x_1 = x_6, x_3 \neq x_6, \underline{x_5 \neq x_4}, x_5 \neq x_7; x_1 = a, x_2 = n, x_3 = d, x_4 \in \{\underline{e},r\}, x_5 = e, x_6 = a, x_7 \in \{e,s\} \rangle$$

DISEQUALITY 4

$$\langle x_1 = x_6, x_3 \neq x_6, \underline{x_5 \neq x_7}; x_1 = a, x_2 = n, x_3 = d, x_4 = r, x_5 = e, x_6 = a, x_7 \in \{e,s\} \rangle$$

DISEQUALITY 4

$$\langle x_1 = x_6, \underline{x_3 \neq x_6}; x_1 = a, x_2 = n, x_3 = d, x_4 = r, x_5 = e, x_6 = a, x_7 = s \rangle$$

DISEQUALITY 2

$$\langle x_1 = x_6; x_1 = a, x_2 = n, x_3 = d, x_4 = r, x_5 = e, x_6 = a, x_7 = s \rangle$$

b)

$$\langle x_1 + 2x_2 - 3x_3 \le 4; x_1 \in [1..10], x_2 \in [2..20], x_3 \in [-10..10] \rangle$$

$$\alpha_1 := \frac{b - \Sigma_{i \in POS-\{1\}} a_i l_i + \Sigma_{i \in NEG} a_i h_i}{a_1} = \frac{4 - 2 \cdot 2 + 3 \cdot 10}{1} = 30$$

$$\mathrm{l}_1' = 1, \mathrm{h}_1' = \min(10, \lfloor 30 \rfloor) = 10$$

$$\alpha_2 := \frac{b - \Sigma_{i \in POS-\{2\}} a_i l_i + \Sigma_{i \in NEG} a_i h_i}{a_2} = \frac{4 - 1 \cdot 1 + 3 \cdot 10}{2} = 16.5$$

$$\mathrm{l}_2' = 2, \mathrm{h}_2' = \min(20, \lfloor 16.5 \rfloor) = 16$$

$$\beta_3 := \frac{-b + \Sigma_{i \in POS} a_i l_i - \Sigma_{i \in NEG-\{3\}} a_i h_i}{a_3} = \frac{-4 + 1 \cdot 1 + 2 \cdot 2 - 0}{3} = 0.3333$$

$$\mathrm{l}_3' = \max(-10, \lceil 0.3333 \rceil) = 1, \mathrm{h}_3' = 10$$

$$\langle x_1 + 2x_2 - 3x_3 \le 4; x_1 \in [1..10], x_2 \in [2..16], x_3 \in [1..10] \rangle$$

2.

a)

(i) $x_1 \wedge (x_2 \vee x_3) = x_4$

$x_1 \wedge y = x_4, x_2 \vee x_3 = y$

(ii) $\neg(x_1 \wedge (x_2 \wedge x_3)) = x_4$

$\neg y = x_4, x_1 \wedge (x_2 \wedge x_3) = y$

$\neg y = x_4, x_1 \wedge z = y, x_2 \wedge x_3 = z$

(iii) $(x_1 \vee (x_2 \wedge x_3)) \vee x_4 = x_5$

$y \vee x_4 = x_5, x_1 \vee (x_2 \wedge x_3) = y$

$y \vee x_4 = x_5, x_1 \vee z = y, x_2 \wedge x_3 = z$

b)

$$\langle (x_1 \wedge x_2) \vee (x_2 \wedge x_3) = x_4, \neg x_1 \wedge (x_5 \vee x_6) = x_7 \wedge (x_2 \wedge x_3);$$
$$x_1 \in \{0,1\}, x_2 = 1, x_3 = 1, x_4 \in \{0,1\}, x_5 = 1, x_6 \in \{0,1\}, x_7 = 1\rangle$$

First the constraints must be transformed to simple form:

$$\{(x_1 \wedge x_2) \vee (x_2 \wedge x_3) = x_4, \neg x_1 \wedge (x_5 \vee x_6) = x_7 \wedge (x_2 \wedge x_3)\}$$

$$\{y_1 \vee (x_2 \wedge x_3) = x_4, x_1 \wedge x_2 = y_1, \neg x_1 \wedge (x_5 \vee x_6) = y_3, x_7 \wedge (x_2 \wedge x_3) = y_3\}$$

$$\{y_1 \vee y_2 = x_4, x_2 \wedge x_3 = y_2, x_1 \wedge x_2 = y_1, \neg x_1 \wedge y_4 = y_3, x_5 \vee x_6 = y_4,$$
$$x_7 \wedge y_6 = y_3, x_2 \wedge x_3 = y_6\}$$

$$\{y_1 \vee y_2 = x_4, x_2 \wedge x_3 = y_2, x_1 \wedge x_2 = y_1, y_5 \wedge y_4 = y_3, \neg x_1 = y_5, x_5 \vee x_6 = y_4,$$
$$x_7 \wedge y_6 = y_3, x_2 \wedge x_3 = y_6\}$$

Each new variable has the domain {0, 1}. If a variables domain is {0, 1}, it is not marked in the following CSPs.

$$\langle y_1 \vee y_2 = x_4, \underline{x_2 \wedge x_3 = y_2}, x_1 \wedge x_2 = y_1, y_5 \wedge y_4 = y_3, \neg x_1 = y_5, x_5 \vee x_6 = y_4,$$
$$x_7 \wedge y_6 = y_3, x_2 \wedge x_3 = y_6; \underline{x_2 = 1}, \underline{x_3 = 1}, x_5 = 1, x_7 = 1\rangle$$

AND 1

$$\langle y_1 \vee y_2 = x_4, x_1 \wedge x_2 = y_1, y_5 \wedge y_4 = y_3, \neg x_1 = y_5, \underline{x_5 \vee x_6 = y_4}, x_7 \wedge y_6 = y_3,$$
$$x_2 \wedge x_3 = y_6; x_2 = 1, x_3 = 1, \underline{x_5 = 1}, x_7 = 1, y_2 = 1\rangle$$

OR 1

$$\langle y_1 \vee y_2 = x_4, x_1 \wedge x_2 = y_1, y_5 \wedge y_4 = y_3, \neg x_1 = y_5, x_7 \wedge y_6 = y_3, x_2 \wedge x_3 = y_6;$$
$$x_2 = 1, x_3 = 1, x_5 = 1, x_7 = 1, y_2 = 1, y_4 = 1\rangle$$