T-79.192 Special Course in Theoretical Computer Science, 2001
Home assignment 6

return date: Thu Nov 8

1. Let's look at the quantum circuit in course book Figure 9.7 (p. 203). It consists of two gates, both operating on two qubits. The first gate (G_1) applies rotation operator

$$L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

to the first input qubit ($q1$), and leaves the other ($q2$) intact. The second gate G_2 is the XOR gate

$$XOR = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

The inputs to the circuit should both be $|0\rangle$. Assume, however, that in the preparation of $q1$ some error was introduced, so that if you would measure $q1$, you would get "1" with 10 % probability. $q2$ is assumed to be $|0\rangle$ without any error.

a) What is the unitary matrix implementing G_1?
b) Compute the intermediate result (after G_1 has operated) and the final result (after G_1 and G_2) for both the erroneous and the faultless input qubits.
c) It is said in the book (Ch. 5.1) that the error should remain constant. Do you results of b) confirm this?

2. The Hamiltonian for a two-gate Feynman computer (course book p. 86) is

$$H = c_2 \cdot a_1 \cdot M_1 + c_3 \cdot a_2 \cdot M_2 + (c_2 \cdot a_1 \cdot M_1)^\dagger + (c_3 \cdot a_2 \cdot M_2)^\dagger$$

What is the effect of the following errors on the possible states of the cursor system? (α, β and γ are nonzero real numbers.)

a) In term $c_2 \cdot a_1 \cdot M_1$, an erroneous annihilation matrix a' is used for building a_1, instead of the correct matrix a. Other matrices have no errors.

$$a' = \begin{pmatrix} 0 & \alpha \\ 0 & \beta \end{pmatrix}$$

(In the conjugate term, a correct a is used).

b) In term $c_2 \cdot a_1 \cdot M_1$, c' is used instead of the correct creation matrix. Other matrices have no errors.

$$c' = \begin{pmatrix} 0 & \gamma \\ 1 & 0 \end{pmatrix}$$

(again, the corresponding conjugate term can be assumed to use correct operators).

c) Can these errors lead to a situation, where the result is measured and thought to be the final result, althought it isn't?