T-79.192 Special Course in Theoretical Computer Science, 2001
Home assignment 4

return date: Thu 25 Oct

This round is likely to require more work than the previous home assignments. To compensate this, this round will also have double weight when the exercises are graded.

Let's study the behavior of Feynman's quantum computer consisting of two $\sqrt{NOT}$ gates (just the same kind of device as in the course book chapter 4). The initial state of the computer is $|1000\rangle$, i.e. the cursor is in the first of the program counter qubits and the register qubit is in state $|0\rangle$. At time $t = 0$, the system is put into the initial state and then let to evolve for some time T, without any measurements.

Plot, as a function of time, the probability of finding the cursor at the third program counter qubit. Choose the length of the time interval T suitably to give a good idea of how the probability behaves.

This can be done, for example, by solving the differential equation system for the computer, or, if you get allergic reactions from solving equations, by computing successive values of the state vector by using the evolution operator $U(t)$. Whatever method you use, you may use value 1 for the constant h. Using some mathematical application program is recommended (it will save you a lot of work!).

As the result you should return:

1. The picture of how the probability behaves.
2. A brief explanation of how you produced your picture. All details are not needed; this is mainly to help the assistant to see what went wrong and how much consolation points you should get in that unlikely case that there would be something wrong with your solution.
3. Answers to the following questions:
   a) What would have changed if you had used the correct value of h instead of $h = 1$?
   b) Would your picture have changed, and how, if the initial state had been $|1001\rangle$?
   c) To get the final result out of the computer, its state must be measured every now and then until the computer is found in a state where the computation is ready. At which moment t would you recommend the first measurement to be made, and why?