T-79.192 Special Course in Theoretical Computer Science, 2001
Home assignments 3

return date: Thu 18 Oct

1. a) Determine $|\phi\rangle = |\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle$ for $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$.
   b) What is the probability to get result "101" when all qubits of state $|\phi\rangle$ are measured?
   c) Is it possible to know the states of all three qubits of $|\phi\rangle$ after only one or two have been measured? If yes, in which situations (i.e. which qubits must be measured and what the measurement results must be)?

2. Let $|\psi\rangle = \frac{1}{2}|001\rangle + \frac{1}{2}|100\rangle + \frac{1}{2}|010\rangle + \frac{1}{2}|111\rangle$.
   a) Is it possible to know the states of all three qubits of $|\psi\rangle$ after only one or two have been measured? If yes, in which situations (i.e. which qubits must be measured and what the measurement results must be?)
   b) Assume that the third (the rightmost) qubit is measured, and that the measurement result is "1". Into which state does $|\psi\rangle$ collapse after this measurement?
   c) Did $|\psi\rangle$ collapse into a pure state? A state is called *pure* if it can be represented as a tensor product of single qubit states.

3. Let $|0'\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1'\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.
   The vector
   $$|\psi'\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$
   is represented in the basis $\{|0'\rangle, |1'\rangle\}$. What is the probability of getting "1" as result, if we measure $|\psi'\rangle$ in the basis $\{|0\rangle, |1\rangle\}$?