T-79.192 Special Course in Theoretical Computer Science, 2001
Home assignments 2

return date: Thu 4 Oct

1. Calculate eigenvectors and eigenvalues of
$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

2. Tensor product (direct product, Kronecker product) of two matrices is
$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & & \vdots \\ a_{n1}B & a_{n2}B & \dots & a_{nn}B \end{pmatrix}$$
(B can be a matrix of any dimensions).

a) Let $a = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$, $c = \left(\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}\right)$, $I = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$. Calculate $a \otimes I \otimes c$.
b) Represent
$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
as a sum of tensor products of 2×2 -matrices.
c) Let $v = \left(\begin{smallmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{smallmatrix}\right)^T$, and let operator M be formed as a tensor product of three smaller dimensional operators:
$$M = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
Apply operator M to vector v, that is, calculate Mv.
(Hint: $(m_1 \otimes m_2 \otimes \cdots \otimes m_n) \cdot (v_1 \otimes v_2 \otimes \cdots \otimes v_n) = (m_1 \cdot v_1) \otimes (m_2 \cdot v_2) \otimes \cdots \otimes (m_n \cdot v_n)$).

3. Let $v(t)$ be a vector of three components ($v_1(t), v_2(t)$ and $v_3(t)$) and $M = \left(\begin{smallmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{smallmatrix}\right)$. The behavior of vector $v(t)$ is described as differential equation
$$\frac{\partial v(t)}{\partial t} = M \cdot v(t).$$
As you probably know, this is just a compact representation for a group of equations, which describe the behavior of the components of $v(t)$. Write out the equations for $v_1(t), v_2(t)$ and $v_3(t)$.

4. To aid in design of the further home assignments: How much time did you spend solving this set of problems?