T-79.192 Special Course in Theoretical Computer Science, 2001
Home assignments 1

1. Let $a = \binom{1}{1}$ and $b = \binom{1}{-1}$.

   a) Normalize a and b into unit vectors a' and b'.
   b) Represent vector $\binom{1}{0}$ as a linear combination of vectors a' and b'.
   c) Are vectors a and b orthogonal? What about a' and b'?

2. Let

$$a = \begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}, \quad b = \begin{pmatrix} 2 & 1-3i \\ 1+3i & 5 \end{pmatrix}.$$

   a) Are a and b Hermitian?
   b) Are a and b unitary?
   c) Can a matrix be Hermitian and unitary at the same time? If no, why not?; if yes, what kind of matrix can be both?

3. Let

$$H = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

   Calculate $U = e^{-iH}$.