Exercise 5.4 Show that every protocol P_i for agent i over the local state space L_i is induced by some program Pg_i.

For every $s \in L_i$ let $state_s$ be a local proposition such that

$$\pi(s')(state_s) = true \quad \text{if and only if} \quad s = s'$$

Let Pg_i be the program that contains a statement

$$\textbf{if } state_s \textbf{ do } a$$

for every state $s \in L_i$ and every action $a \in P_i(s)$. Then, $\mathrm{Pg}_i^{\pi}(s) = P_i$.

Exercise 6.5 Let $\mathcal{G}$ be a set of global states. Show that if P_i is a deterministic protocol, then there is a decision function D such that P_i is compatible with D in all systems $\mathcal{R}$ over $\mathcal{G}$. Show, moreover, that we can take D to be union-consistent.

Set

$$D(S) = P_i\big(r_i(m)\big), \qquad (r,m) \in S$$

for S such that $(r,m) \in S$ and $(r',m') \in S$ implies $P_i\big(r_i'(m')\big) = P_i\big(r_i(m)\big)$. Then,

$$D\big(\mathbf{IS}_i(s_i,\mathcal{R})\big) = D\big(\{(r,m) \mid r_i(m) = s_i\}\big) = P_i(s_i).$$

Hence, P_i is compatible with D.

Assume that $T_1, T_2, \ldots$ is a collection of pairwise disjoint sets of points such that $D(T_j) = a$ for all j, i.e., $(r,m) \in T_j$ implies $P_i\big(r_i(m)\big) = a$. Then, $D(\cup T_j) = a$ and D is union-consistent.