

Testing of Concurrent Systems 2004

Lecture 1
14th Sep 2004

Welcome!

- ▶ This is T-79.190: Testing of Concurrent Systems
- ▶ Lectures from 8 to 10 am, tutorials from 10 to 11 every Tuesday at T3
- ▶ Cancellations and other notes at the web page (go to <http://www.tcs.hut.fi/>)

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Lecturer

- ▶ Antti Huima = me
- ▶ “Special teacher” = not member of HUT staff
- ▶ Work = Vice President of Research and Development at Conformiq Software

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Practical Matters

- ▶ Website contains all important information
- ▶ The news group can be used for discussion, but I will not follow it
- ▶ Lecture notes will be available on the web for “early access”
- ▶ Printed and distributed via the lecture notes print house

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Requirements

- ▶ Pass the course = pass the examination
- ▶ Tutorials do not award extra points
- ▶ Tutorials form part of the requirements for the examination
- ▶ Model answers will be made available

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Tutorials

- ▶ Tutorials from 10 to 11 am, after the lectures
- ▶ Begin next week
- ▶ No tutorial today
- ▶ Subject = lectures of the previous week

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Subject

- ▶ Real subject = formal conformance testing (FCT)
- ▶ “Testing of concurrent systems” is a historical title
- ▶ What is “formal conformance testing”?

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Testing

- ▶ Testing is the process of
 1. interacting with a system, and
 2. evaluating the results, in order to
 3. determine if the system conforms to its specification
- ▶ In testing setup, the system is known as the system under test (SUT)

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Interacting

- ▶ If you can't interact with a system, the system is uninteresting
- ▶ Interacting covers anything you can do with the system

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Conforms

- ▶ Interaction does not imply judgement
- ▶ Conformance = correspondence in form or appearance
- ▶ Conformance to a specification = “works as specified”
- ▶ Were the results of the interaction allowed by the specification?

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Formal

- ▶ Formal = “according to a form”
- ▶ Here: testing is based on a mathematical, formalized foundation
- ▶ Not: testing based on a rigorous process where you need to fill lots of bureaucratic forms
- ▶ Also: “formal methods based”, but this is very vague

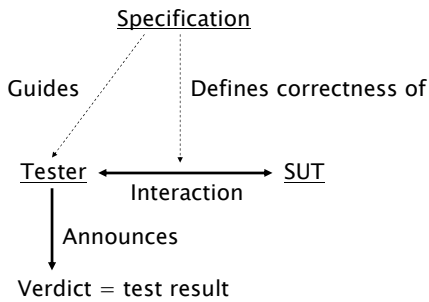
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Operational specification

- ▶ Specifies how a system should work
- ▶ Operational = describes behaviour, not e.g. usability scores
- ▶ Operational: “after 3 s, system must respond with X”
- ▶ Non-operational: “users must like the stuff”
- ▶ From now on just “specification” (assume operational)

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FCT setup



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Tester

- ▶ Tester has two functions:
 - Interact = generate behaviour
 - Give verdict = judge behaviour
- ▶ These two functions can be separated

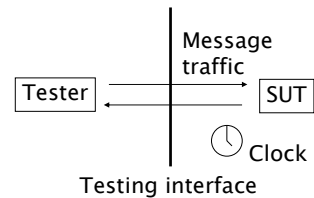
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Verdicts

- ▶ Typical verdicts:
 - PASS = system behaved ok
 - FAIL = system behaved badly
 - ERROR = tester messed up
 - INCONCLUSIVE = PASS, but some important feature was not yet tested

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Testing interface



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Testing interface

- ▶ All interaction happens through the testing interface
- ▶ Bidirectional message passing
- ▶ All transmissions have a time stamp
- ▶ Every event has a distinct time stamp

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Directions

- ▶ Input
 - input to the SUT
 - output from the tester
- ▶ Output
 - output from the SUT
 - input to the tester

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Alphabets

- ▶ Σ_{in} is the set of input messages
- ▶ Σ_{out} is the set of out messages
- ▶ Σ is the union of the two
- ▶ Messages “contain” their direction
- ▶ Alphabet = traditional name for a set of potential messages

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Events

- ▶ Event = message + a time stamp
- ▶ Thus, event = (member of Σ) + (nonnegative real number)
- ▶ Formally, set of events is $\Sigma \times [0, \infty)$
- ▶ E.g. $\langle \text{“hello”}_{in}, 1.4 \text{ s} \rangle$

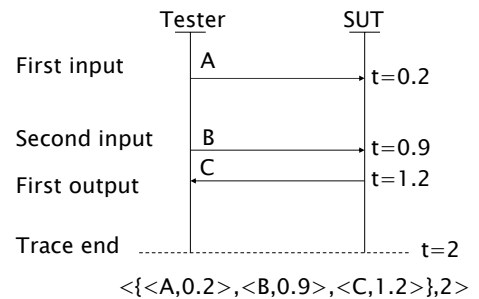
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Traces

- ▶ A trace denotes an observation of events for a certain time
- ▶ Trace = a finite set of events with distinct time stamps + end time stamp
- ▶ E.g. $\langle \{ \langle \text{“hello”}_{in}, 0.5 \rangle \}, 0.8 \rangle$

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Graphical sketch



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lecture 1 summary

- ▶ Testing = interact + judge
- ▶ Specification, tester, SUT
- ▶ Testing interface = point of interaction
- ▶ Trace = a finite series of events observed during a finite time span

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Testing of Concurrent Systems 2004

Lecture 2
14th Sep 2004

Review of previous lecture

- ▶ Testing = interact + judge
- ▶ Specification, tester, SUT
- ▶ Testing interface = point of interaction
- ▶ Trace = a finite series of events observed during a finite time span

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Process notation

- ▶ We need a notation for “computational processes”, i.e. a programming language to describe
 - implementations = SUTs
 - operational specifications as “reference implementations”
 - full testers
 - testing strategies = interaction strategies

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Requirements

- ▶ Support data, time, concurrency
- ▶ Familiar
- ▶ Compact
- ▶ Executable

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The choice

- ▶ Scheme, a dialect of LISP
- ▶ But standard Scheme lacks concurrency and time
- ▶ Solution: extend Scheme slightly

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Introduction to Scheme

- ▶ Scheme is a easy and clean dialect of LISP
- ▶ Scheme = interpreter for applicative order λ -calculus
- ▶ See e.g. Abelson & Sussman

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Standard Scheme

```
(define (factorial n)
  (if (<= n 1)
      1
      (* n (factorial (- n 1)))))
```

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Standard Scheme

```
(define (make-add-to-list n)
  (lambda (ls)
    (map (lambda (x) (+ x n)) ls)))
(let ((z (make-add-to-list 5)))
  (z '(1 2 3)))
```

→ '(6 7 8)

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Extensions

- ▶ We extend Scheme with procedures:
 - spawn
 - make-rendezvous-point
- ▶ A special form:
 - sync

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Spawn

- ▶ Creates a new thread
- ▶ Use: (spawn <thunk>)
- ▶ Returns nothing

```
(spawn (lambda () ...))
```

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Make-rendezvous-point

- ▶ Creates a point of synchronous internal communication
- ▶ Use: (make-rendezvous-point)
- ▶ Returns a new rendezvous point
- ▶ Rendezvous points are used by the (sync ...) form

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Sync

- ▶ General I/O and wait form
- ▶ Use:

```
(sync
  (input <var> <body> ...) ...
  (output <expr> <body> ...) ...
  (read <point> <var> <body> ...) ...
  (write <point> <expr> <body> ...) ...
  (wait <expr> <body> ...) ...)
```

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External I/O

- ▶ (input <var> <body> ...) attempts to read a message from the environment; if successful, store data to <var> and continue with <body> ...
- ▶ (output <expr> <body> ...) attempts to write a message to the environment; if successful, continue with <body> ...

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Internal I/O

- ▶ (read <point> <var> <body> ...) attempts to read a message from point <point>; if successful, store it to <var> and continue with <body> ...
- ▶ (write <point> <expr> <body> ...) attempts to write <expr> to point <point>; if successful, continue with <body> ...

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Timeout

- ▶ (wait <expr> <body> ...) attempts to wait for <expr> seconds; if nothing else happens until that amount of time, continue with <body> ...

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Choice

- ▶ Choice between all items enabled at the same point of time is nondeterministic

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Examples

```
(define (run)
  (sync (input x (run))))

(run)
```

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Zero-time execution principle

- ▶ We assume that all Scheme execution consumes zero time
- ▶ The only exception is waiting at sync

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Examples

```
(define (echo)
  (sync (input x
             (sync (output x (echo))))))

(echo)
```

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Examples

```
(define (echo)
  (sync (input x (wait-and-echo x))))

(define (wait-and-echo x)
  (sync (wait (a-delay)
            (sync (output x (echo))))))

(echo)
```

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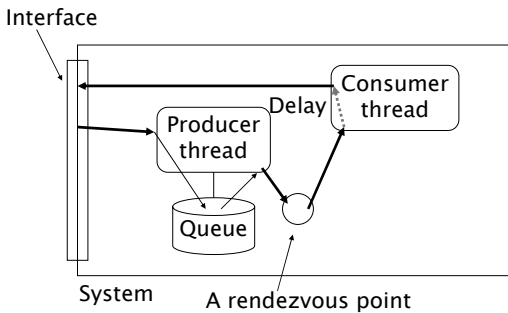
Queued echo

```
(define (run)
  (let ((queue (make-queue))
        (point (make-rendezvous-point)))
    (spawn (lambda ()
              (producer point queue)))
    (spawn (lambda ()
              (consumer point))))))

(run)
```

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Queued echo architecture



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Queued echo

```
(define (consumer point)
  (sync (read point x
          (sync (wait (a-delay)
                    (sync (output x
                          (consumer point))))))))))
```

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Queued echo

```
(define (producer point queue)
  (if (empty-queue? queue)
      (sync (input x (queue-insert! queue x))
          (sync (input x (queue-insert! queue x))
                (write point (queue-front queue)
                          (queue-remove! queue))))
      (producer point queue))
```

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Lecture 2 summary

- ▶ We use Scheme with concurrency extensions to denote processes
 - spawn
 - make-rendezvous-point
 - sync

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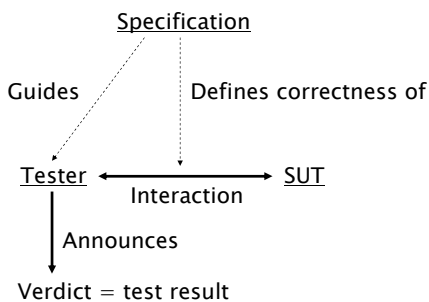
Lecture 3
21th Sep 2004

Course this far

- | | |
|---|--|
| 1 | ▶ Introduction
▶ General concepts
▶ Traces |
| 2 | ▶ Concurrent Scheme |

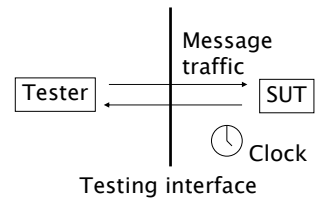
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FCT setup (replay)



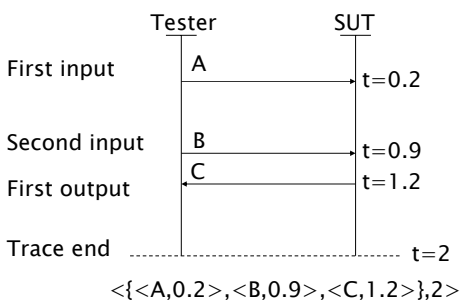
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Testing interface (replay)



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A trace (replay)



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Traces

- ▶ Traces denote finitely long observations on the testing interface
- ▶ A trace contains a finite number of events and an end time stamp
- ▶ Traces are the *lingua franca* for discussing behaviours

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Traces

- ▶ Alphabet
- ▶ Event
- ▶ Trace
- ▶ Trace prefix
- ▶ Empty trace
- ▶ Trace extension
- ▶ Snapshot
- ▶ Difference time

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Alphabets

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- ▶ Formally, set of events is $\Sigma \times [0, \infty)$
- ▶ E.g. $\langle \text{“hello world”}_{in}, 1.4 \text{ s} \rangle$

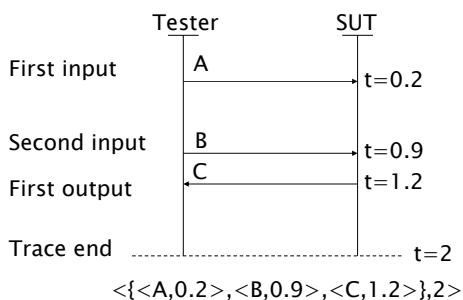
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Graphical sketch



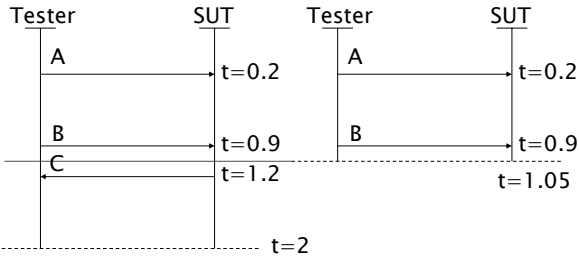
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Prefixes

- ▶ A trace is a prefix of another, if the first trace can be extended in time to become the second one
- ▶ Let T and T' be traces
- ▶ $T = \langle E, t \rangle$ is a prefix of $T' = \langle E', t' \rangle$ (write $T \preceq T'$) if
 - $t \leq t'$ and
 - $E = \{ \langle \alpha, \kappa \rangle \mid \langle \alpha, \kappa \rangle \in E' \wedge \kappa < t \}$

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Prefix sketch



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Empty trace

- ▶ $\langle \emptyset, 0 \rangle$ is the empty trace, denoted by ϵ
- ▶ The empty trace is a prefix of every other trace
- ▶ Empty trace has no information content

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Extensions

- ▶ A trace T is an extension of trace T' if the trace T' is a prefix of trace T
- ▶ Thus, being extension = reverse of being prefix

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Prefix set

- ▶ $\text{Pfx}(T)$ is the set of all prefixes of T
- ▶ $\text{Pfx}(T) = \{ T' \mid T' \preceq T \}$
- ▶ Note: If $T \preceq T'$ then $\text{Pfx}(T) \subseteq \text{Pfx}(T')$

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Snapshot

- ▶ Denote $\Sigma_\tau = \Sigma \cup \{\tau\}$
- ▶ Here τ is an object that does not belong to set Σ
- ▶ Let $T = \langle E, t \rangle$
- ▶ Assume $\kappa < t$
- ▶ Denote by $T|_\kappa$ the event at time κ , or τ if no event at trace T has time stamp κ

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Snapshot example

- ▶ Suppose $T = \langle \langle \langle A, 1 \rangle \rangle, 2 \rangle$
- ▶ $T|_1 = A$
- ▶ $T|_{1.5} = \tau$
- ▶ $T|_2$ is not defined
- ▶ $T|_3$ is not defined

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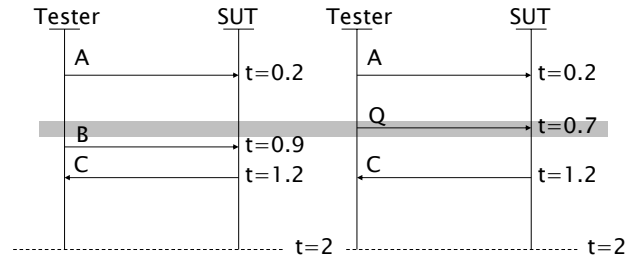
Difference time

- ▶ Suppose T and T' are traces such that T is not a prefix of T' and T' is not a prefix of T
- ▶ T and T' are hence not equal
- ▶ Define

$$\Delta(T, T') = \min t^* : T|_{t^*} \neq T'|_{t^*}$$

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Difference time sketch



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Specifications

- ▶ Set of specifications
- ▶ Set of valid traces
- ▶ Prefix-completeness
- ▶ Seriality

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Set of specification

- ▶ S is a countable set of specifications
- ▶ Could be e.g.
 - Set of syntactically correct UML state charts
 - Set of valid English documents
- ▶ Structure not relevant
- ▶ Assume exists

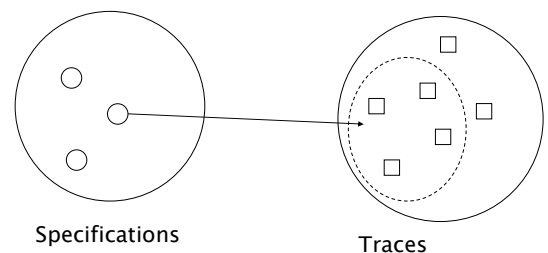
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Valid traces

- ▶ Every specification denotes a set of traces: the set of *valid traces*
- ▶ If S is a specification, $\text{Tr}(S)$ is the set of valid traces for S
- ▶ $\text{Tr}(S)$ must contain ϵ
- ▶ $\text{Tr}(S)$ must be *prefix-complete*
- ▶ $\text{Tr}(S)$ must be *serial*

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Specifications



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Prefix-completeness

- ▶ A set X of traces is prefix-complete if the following holds:
- ▶ If $T \in X$ and $T' \preceq T$ then also $T' \in X$
- ▶ If a trace belongs to a prefix-complete set, then also all its prefixes belong to the set
- ▶ Why $\text{Tr}(S)$ must be prefix-complete?

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Motivation for prefix-completeness

- ▶ $\text{Tr}(S)$ denotes a set of *acceptable behaviours*
- ▶ Assume T is an acceptable behaviour
- ▶ Can you imagine a case where T' , a prefix of T , would be not acceptable?

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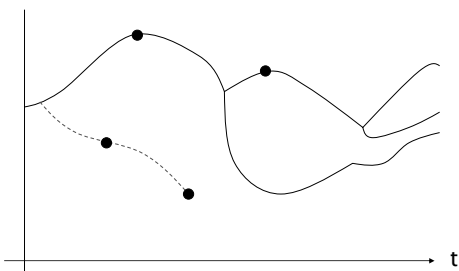
Lecture 4
21th Sep 2004

Seriality

- ▶ A set X of traces is *serial* if for every $\langle E, t \rangle \in X$ and for every $\delta > 0$ this holds:
- ▶ There exists $\langle E', t + \delta \rangle \in X$ such that $\langle E, t \rangle \preceq \langle E', t + \delta \rangle$
- ▶ Every trace of X has at least one arbitrarily long extension in X

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Seriality sketch



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Motivation for seriality

- ▶ Suppose non-serial $\text{Tr}(S)$
- ▶ There exists a valid trace T without an extension
- ▶ Let $T' \preceq T$
- ▶ Is the behaviour T' acceptable?
- ▶ Why? And why not?

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Implementations

- ▶ We assume there exists a countable set of implementations, denoted by \mathbb{I}
- ▶ Could be e.g.
 - Set of all valid JAVA programs
 - Set of all valid C programs
 - Set of all functioning digital circuits

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Failure model

- ▶ Failure model links a specification to its potential implementations
- ▶ A failure model is a function $\mu: \mathbb{S} \rightarrow (\mathbb{I} \rightarrow [0, 1])$
- ▶ For every $s \in \mathbb{S}$, it holds that $\sum_i \mu(s)[i] = 1$
- ▶ Hence $\mu(s)$ is a discrete probability distribution over implementations

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Use of failure models

- ▶ Failure model is a hypothesis about implementations and their potential defects
- ▶ Example: Boundary Value Pattern and the related failure model

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Testing strategies

- ▶ A testing strategy is a strategy on how to interact with an implementation
- ▶ Let \mathbb{T} denote the countable set of all testing strategies
- ▶ What happens when a testing strategy is executed “against” an implementation?

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Execution

- ▶ Testing strategy + implementation yields a sequence of traces T_1, T_2, T_3, \dots
- ▶ Here $T_1 \preceq T_2 \preceq T_3 \preceq \dots$
- ▶ These correspond to *test steps*
- ▶ Many different trace sequences are possible
- ▶ How do we formalize this?

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Semantic function ξ

- ▶ Maps implementation, testing strategy and “system state” to extensions of the currently observed trace
- ▶ Actually to a probability distribution of extensions
- ▶ System state = trace observed this far

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ξ function properties

- ▶ Gives a probability distribution
- ▶ Test steps are proper trace extensions
- ▶ Progressivity
- ▶ Test steps are disjoint

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Signature

- ▶ Let \mathcal{T} denote the set of all traces
- ▶ The signature is
$$\xi : \mathbb{I} \times \mathcal{T} \times \mathcal{T} \rightarrow (\mathcal{T} \rightarrow [0, 1])$$

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Gives probability distribution

- ▶ For all i, s and T , it must hold that
$$\sum_{T'} \xi(i, s, T)[T'] = 1$$

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Test steps = proper trace extensions

- ▶ For all i, s, T and T' it must hold that
$$\xi(i, s, T)[T'] > 0 \Rightarrow T < T'$$
- ▶ Hence: every test step consumes time

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Test step disjointness

- ▶ For any i, s, T , and T_1 and T_2 it must hold that if $T_1 \neq T_2$ and
- ▶ $\xi(i, s, T)[T_1] > 0$ and
- ▶ $\xi(i, s, T)[T_2] > 0$, then
- ▶ $T_1 \not\prec T_2$, and
- ▶ $T_2 \not\prec T_1$
- ▶ A technical convenience

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Progressivity

- ▶ There does not exist an infinite sequence T_1, T_2, T_3, \dots and a constant $K \in \mathbb{R}$ such that
- ▶ $\xi(i, s, T_i)[T_{i+1}] > 0$ for all i , but such that for all $T_i = \langle E_i, t_i \rangle$ it holds that $t_i < K$.

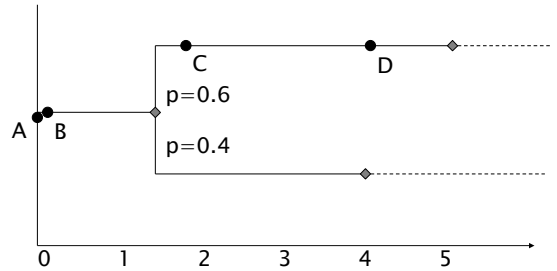
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Trace probabilities

- ▶ Let $P[i, s, T]$ denote the probability of observing T as a prefix of a long enough trace when strategy s is executed against implementation i
- ▶ Idea is to compute the product of the preceding test step probabilities
- ▶ Multiply this with the probabilities of those test steps that produce extensions of trace T
- ▶ Technical definitions in the handouts

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Execution sketch



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Testing of Concurrent Systems 2004

Lecture 5
28th Sep 2004

Course this far

14.9	1	▶ Introduction ▶ General concepts ▶ Traces
	2	▶ Concurrent Scheme
21.9	3	▶ Traces, specifications
	4	▶ Seriality, execution introduction

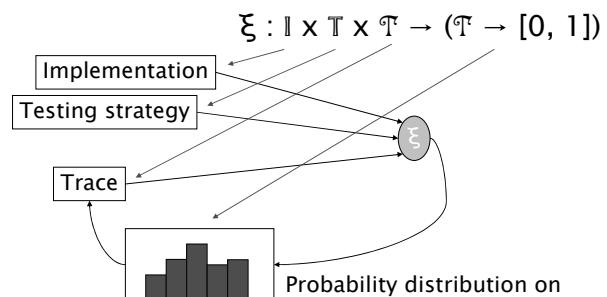
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Traces and specifications

- ▶ Trace = set of events + end time stamp
 - Event = message + time stamp
 - Prefix, extension, snapshot
- ▶ Specification \cong set of valid traces
 - Prefix-closed
 - Serial

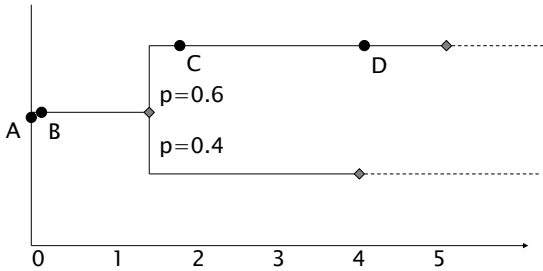
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Execution



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Execution sketch

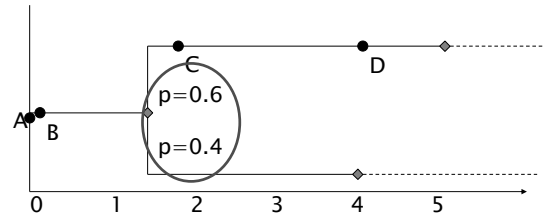


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Gives probability distribution

- ▶ For all i, s and T , it must hold that

$$\sum_{T'} \xi(i, s, T)[T'] = 1$$



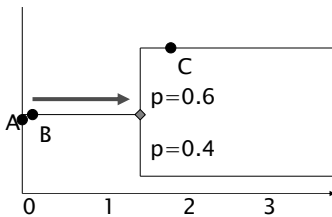
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Test steps = proper trace extensions

- ▶ For all i, s, T and T' it must hold that

$$\xi(i, s, T)[T'] > 0 \Rightarrow T < T'$$

- ▶ Hence: every test step consumes time



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Test step disjointness

- ▶ For any i, s, T , and T_1 and T_2 it must hold that if $T_1 \neq T_2$ and
- ▶ $\xi(i, s, T)[T_1] > 0$ and
- ▶ $\xi(i, s, T)[T_2] > 0$, then
- ▶ $T_1 \not< T_2$, and
- ▶ $T_2 \not< T_1$
- ▶ A technical convenience

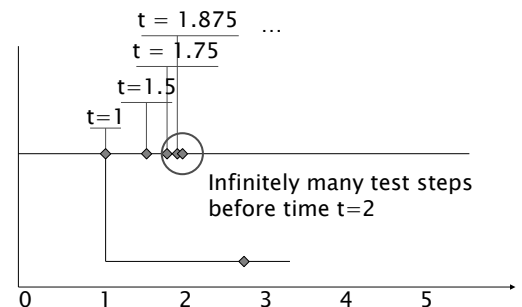
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Progressivity

- ▶ There does not exist an infinite sequence T_1, T_2, T_3, \dots and a constant $K \in \mathbb{R}$ such that
- ▶ $\xi(i, s, T_i)[T_{i+1}] > 0$ for all i , but such that for all $T_i = \langle E_i, t_i \rangle$ it holds that $t_i < K$.

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Non-progressivity sketch



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Choice of ξ

- ▶ We have defined properties of ξ , not the function itself
- ▶ The particular choice for ξ depends on
 - the set of implementations I ,
 - the set of testing strategies T , and
 - the desired structure of test steps.

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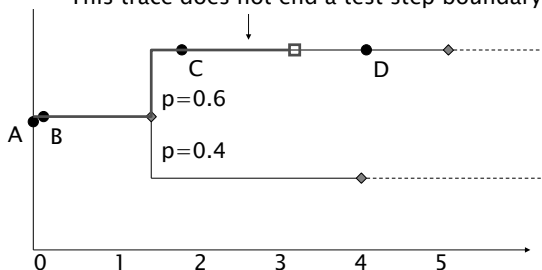
Trace probabilities

- ▶ Random experiment:
- ▶ An implementation i and a testing strategy s have been chosen
- ▶ A trace prefix T^* has been fixed, $T^* = \langle E, K \rangle$
- ▶ s is executed against i many times, yielding traces $T_1 = \langle E_1, t_1 \rangle$, $T_2 = \langle E_2, t_2 \rangle$, ..., such that for all n , $t_n > K$
- ▶ What is the probability that for a uniformly chosen n , $T_n[K] = T^*$?
 - $X[t]$ is that prefix of X whose end time stamp is t

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Problem

This trace does not end a test step boundary



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Solution

- ▶ Traces that end at test step boundaries are easy: compute product probability
- ▶ Traces that end at non-boundaries require an extra construct

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Step 1: traces at test step boundaries

- ▶ Denote by $P^*[i, s, T]$:

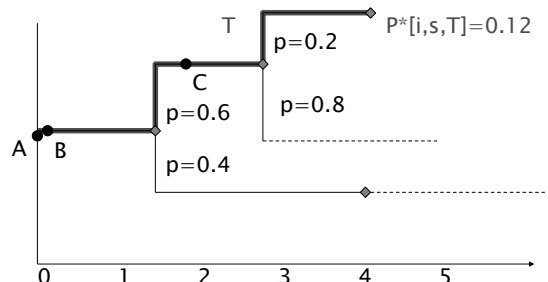
$$\max_{T_1, \dots, T_n} \prod_{i \in [1, n-1]} \xi(i, s, T_i)[T_{i+1}]$$

where $T_1 = \epsilon$ and $T_n = T$

- ▶ $P^*[i, s, T]$ is the compound probability for trace T , if T "happens" at test step boundary

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Sketch



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Step 2: traces at non-boundaries

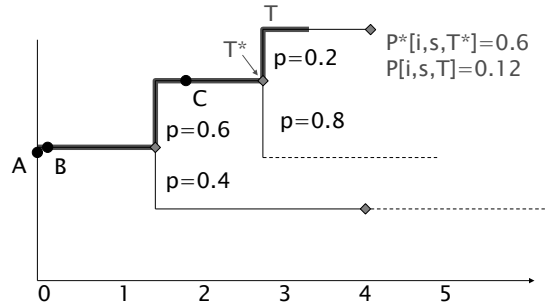
- Denote by $P[i,s,T]$

$$P^*[i,s,T^*] \times \left(\sum_{T': T \leq T'} \xi(i,s,T^*)(T') \right)$$

- T^* is largest prefix of T such that $P^*[i,s,T^*] > 0$

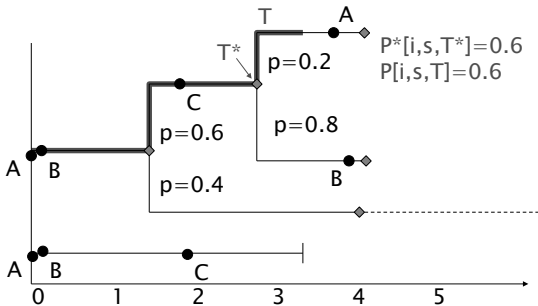
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Sketch



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Sketch



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Sanity checks

- If $P^*[i,s,T] > 0$,
 - then $P[i,s,T] = P^*[i,s,T]$.
 - Ok.
- If $P[i,s,T] = 0$ (trace T cannot be produced),
 - there still exists the greatest prefix T^* of T such that $P^*[i,s,T^*] > 0$.
 - Every test step succeeding T^* must result in a trace differing from T — ok.

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Sanity check 1 memo

- Assume $P^*[i,s,T] > 0$
- Note $T^* = T$
- $P[i,s,T] =$

$$P^*[i,s,T] \times \left(\sum_{T': T \leq T'} \xi(i,s,T)(T') \right)$$

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Execution summary

- ξ defines execution semantics
- Properties for ξ
 - Gives probability distribution over traces
 - Test step = trace extension
 - Test step disjointness
 - Progressivity
- However, no concrete structure
- $P[i,s,T]$ is the probability of producing trace T when s is run against i
 - Hides test steps

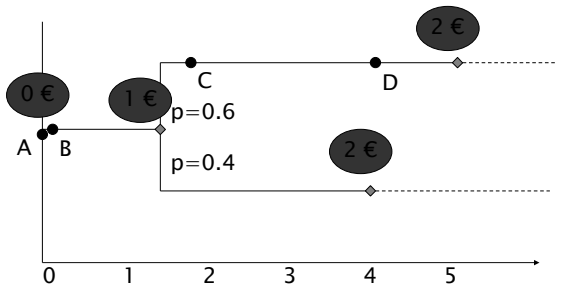
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Why test steps?

- ▶ 1 test step =
 - unit of testing cost
 - unit of benefit
- ▶ Testing can be stopped between test steps, but not during them
 - stopping criteria
- ▶ Technical construct for describing arbitrarily long executions without the concept of “an infinite trace” (there is no such concept here)

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Cost or benefit



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Measuring the “size” of a trace

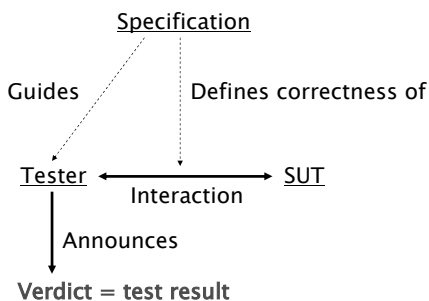
- ▶ Temporal length = end time stamp
- ▶ Size of event set
- ▶ Number of test steps used to produce

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Testing of Concurrent Systems 2004

Lecture 6
28th Sep 2004

Verdicts



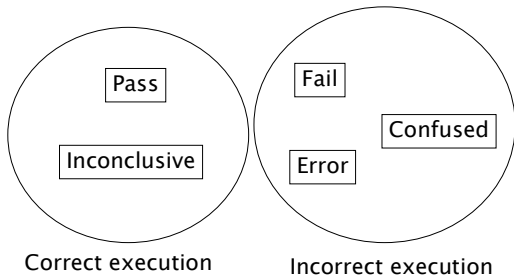
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Verdicts

- ▶ Pass
- ▶ Fail
- ▶ Error
- ▶ Inconclusive
- ▶ Confused

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Verdicts



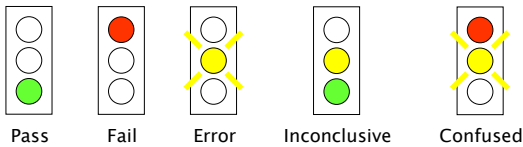
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Verdicts explained

Verdict	Explanation
Pass	System under test has behaved correctly
Fail	System under test has behaved incorrectly
Error	Tester has behaved incorrectly
Inconclusive	Pass, but a certain test purpose has not been met
Confused	Fail-and-Error, result produces by an ambiguous specification (a special corner case)

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Verdicts as traffic lights



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Calculating verdict

- ▶ Verdict is calculated from a trace T and a specification S

$$\text{verdict}(T,S) \in \{ \text{pass}, \text{fail}, \text{error}, \text{conf} \}$$

- ▶ No inconc, because requires a test purpose

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Pass verdict

$$\text{verdict}(T,S) = \text{pass}$$

if and only if

$$T \in \text{Tr}(S)$$

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Other verdicts

- ▶ Hence, $T \notin \text{Tr}(S)$ implies

$$\text{verdict}(T,S) \in \{ \text{fail}, \text{error}, \text{conf} \}$$

- ▶ There is one verdict for $T \in \text{Tr}(S)$, and three for the other case

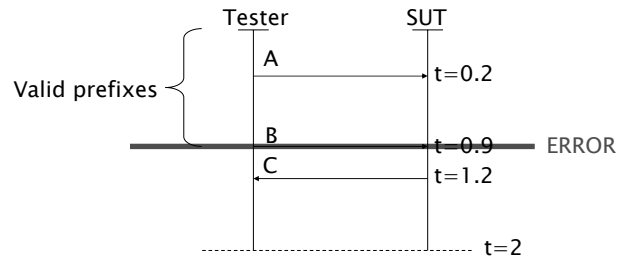
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Other verdicts

- ▶ The problem: how to classify the cases $T \notin \text{Tr}(S)$ into
 - errors of the SUT (\rightarrow fail),
 - errors of the tester (\rightarrow error),
 - and those cases where the erring party cannot be defined (\rightarrow confused)?

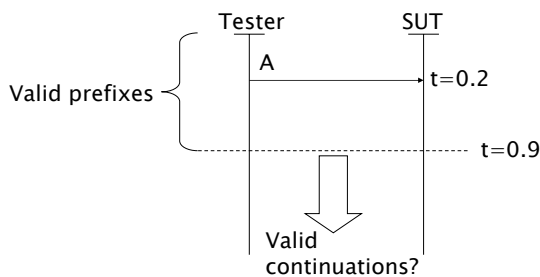
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Solution sketch (1)



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Solution sketch (2)



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Solution sketch (3)

- ▶ All valid continuations differ from T first at input events?
 - \rightarrow error
- ▶ All valid continuations differ from T first at output events?
 - \rightarrow fail
- ▶ Otherwise
 - \rightarrow confused

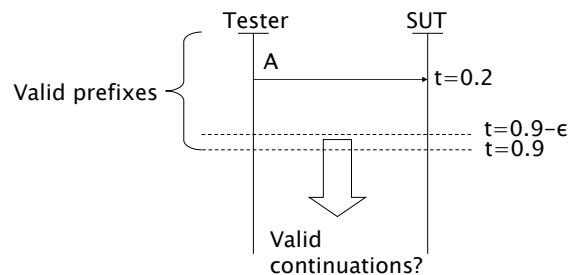
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Technicality

- ▶ The set of end time stamps for the valid prefixes of T can be either open or closed at the upper boundary
- ▶ Open set requires basically a limit construct (as usual)

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Solution sketch (4)



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Details

- ▶ Assume $T \notin \text{Tr}(S)$
- ▶ Let $V = \text{Tr}(S) \cap \text{Pfx}(T)$
 - Note: $\epsilon \in V$
- ▶ Let $K = \{ t \mid \exists E: \langle E, t \rangle \in V \}$
- ▶ K is either
 - closed: $[0, t]$, or
 - open: $[0, t)$.

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Example (closed set)

- ▶ $\text{Tr}(S) =$
 $\cup \{ \text{Pfx}(\langle \langle A, t' \rangle, t \rangle) \mid t \in [2, \infty), t' \leq 1 \}$
- ▶ $T = \langle \emptyset, 10 \rangle$
- ▶ Note that $T \notin \text{Tr}(S)$
- ▶ $V = \{ \langle \emptyset, t \rangle \mid t \leq 1 \}$
- ▶ $K = [0, 1]$
- ▶ Especially $\langle \emptyset, 1 \rangle$ is in V , because $\langle \langle A, 1 \rangle, 1.1 \rangle$ is valid

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Example (open set)

- ▶ $\text{Tr}(S) =$
 $\cup \{ \text{Pfx}(\langle \langle A, t' \rangle, t \rangle) \mid t \in [2, \infty), t' < 1 \}$
- ▶ $T = \langle \emptyset, 10 \rangle$
- ▶ Note that $T \notin \text{Tr}(S)$
- ▶ $V = \{ \langle \emptyset, t \rangle \mid t < 1 \}$
- ▶ $K = [0, 1)$
- ▶ Especially $\langle \emptyset, 1 \rangle$ is not in V , because for any $t' < 1$, event $\langle A, t' \rangle$ should belong to the event set at time 1

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Details continued

- ▶ Choose $\delta \in K$ (note: $0 \in K$ always, so K is not empty)
- ▶ Let X_δ denote the set of all valid extensions of $T[\delta]$ beyond the end time stamp of T

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Details continued

- ▶ For every T' in X_δ , T' differs from T and $\Delta(T, T')$ is defined
- ▶ For every T' , denote by $\alpha T|_{\Delta(T, T')}$ if not τ
 - Otherwise denote by $\alpha T'|_{\Delta(T, T')}$
 - Note: α can not be τ
- ▶ Let D_δ be the union of all α
- ▶ D_δ lists those events on which valid extensions of $T[\delta]$ differ from T

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Details continued

- ▶ Assume there exists $\delta \in K$ such that $D_\delta \subseteq \Sigma_{in}$
 - Tester failure \rightarrow error
- ▶ Assume there exists $\delta \in K$ such that $D_\delta \subseteq \Sigma_{out}$
 - SUT failure \rightarrow fail
- ▶ Otherwise
 - undefined \rightarrow confused

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Details continued

- ▶ If K is closed, we can always choose $\delta = \max K$
- ▶ If K is open, we must choose a δ "close enough" the upper bound of K
 - $(\sup K) - \epsilon$ for $\epsilon > 0$

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Disjointness

- ▶ $D_\delta \subseteq \Sigma_{in}$ and $D_\delta \subseteq \Sigma_{out}$ are disjoint conditions, because
 - $\delta \leq \epsilon$ implies $D_\epsilon \subseteq D_\delta$
 - D_δ is always non-empty
 - Σ_{in} and Σ_{out} are disjoint

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Summary

- ▶ Is $T \in \text{Tr}(S)$?
 - Verdict is "pass"
- ▶ Else
 - Does there exists $\delta \in K$ such that $D_\delta \subseteq \Sigma_{out}$?
 - Verdict is "fail"
 - Otherwise, does there exists $\delta \in K$ such that $D_\delta \subseteq \Sigma_{in}$?
 - Verdict is "error"
 - Otherwise verdict is "confused"

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Testing of Concurrent Systems 2004

Lecture 7
12th Oct 2004

Course this far

14.9	1	▶ Introduction ▶ General concepts ▶ Traces
	2	▶ Concurrent Scheme
21.9	3	▶ Traces, specifications
	4	▶ Seriality, execution introduction
28.9	5	▶ Test steps and execution
	6	▶ Test verdicts

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"Scheme in everything"

- ▶ Until now, testing strategies and specifications have not had a structure
- ▶ We now consider Scheme programs as
 - implementations,
 - testing strategies,
 - testers, and
 - specifications.

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Structure of a complete program

```
;; Definitions  
(define ...)  
(define ...)  
(define ...)  
...  
;; Entry point  
(expr ...)
```

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Programs that generate behaviour

- ▶ Programs denote computational processes
- ▶ A computational process is characterized by its external behaviour, i.e. traces
- ▶ But we already have a function for generating traces, namely ξ

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Behaviour through ξ

- ▶ Suppose i is a Scheme program
- ▶ One way to characterize the behaviour of the program i is the set
$$ETr(i) = \{ T \mid \exists s \in \mathbb{T} : P[i, s, T] > 0 \}$$
- ▶ This is the set of all traces that are produced by *some* testing strategy with a non-zero probability
- ▶ Here we assumed that ξ was defined for Scheme programs

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Defining ξ

- ▶ We assume a definition of ξ with the following properties:
 - Invalid input causes output ERR and program termination
 - ERR is a special symbol we reserve for this purpose
 - Division by zero or other run-time error causes output ERR and program termination
 - Termination of last thread \rightarrow program termination
 - A terminated program does not produce any output whatsoever
 - Otherwise, assumed semantics of Scheme are preserved

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Testing strategies (general)

- ▶ We assume that the set of testing strategies \mathbb{T} contains at least all fixed input message sequences (a reasonable assumption).
- ▶ Hence, $ETr(i)$ is *the* set of traces that a program can produce “against a suitable environment”.

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Example

- ▶ Consider this program p :
(sync)
- ▶ $ETr(p)$ contains traces void of events, and every trace that contains ERR and one or more input messages

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Another example

- ▶ Consider this program q:

```
(define (run)
  (sync (input x (run))))
(run)
```

- ▶ ETr(q) contains all traces with only input messages; no ERR output is possible.

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Summary of ETr(p)

- ▶ Set of traces that program p can generate with non-zero probability against at least one environment = testing strategy
- ▶ Invalid input or invalid computation causes program to halt → ERR, then no output

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Programs as testing strategies

- ▶ Programs function as testing strategies “as implementations”
- ▶ We do not consider strategies that can crash
- ▶ We do not give more rigorous definition (at least now)
- ▶ Semantics implemented by ξ (as usual)

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Programs as full testers

- ▶ Assume the existence of verdict-announcing functions:
 - (pass)
 - (fail)
- ▶ That’s it!

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Example

- ▶ Tester for Echo Program:

```
(begin
  (sync (output “hello”
    (sync (input x
      (if (equal? x “hello”)
        (pass)))
      (wait 1 #f))))
  (fail))
```

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Programs as specifications

- ▶ We have seen how a program can be used as an implementation
- ▶ We now turn to consider how a program can be used as a specification

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Properties of specifications

- ▶ Suppose S is any specification
- ▶ Then $\text{Tr}(S)$ is
 - prefix-complete
 - serial
 - non-empty

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Interpreting programs as specifications

- ▶ A program is interpreted as a specification by considering it as a *reference implementation*
- ▶ Any behaviour that the reference implementation can produce is valid
- ▶ Any behaviour that the reference implementation could not produce is invalid
- ▶ Hence, what is $\text{Tr}(p)$ for a program p ?

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What is $\text{Tr}(p)$

- ▶ Is $\text{Tr}(p) = \text{ETr}(p)$?
 - No
 - $\text{ETr}(p)$ contains traces where the system has halted due to an execution error (output ERR has been produced)
 - Execution error could be caused by invalidly received input
 - A trace that contains invalid input cannot be included in the set of valid traces!

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Fix 1

- ▶ Let $\text{VTr}(p)$ be that subset of $\text{ETr}(p)$ that does not include ERR outputs:
$$\text{VTr}(p) = \{ \langle E, t \rangle \mid \langle E, t \rangle \in \text{ETr}(p) \wedge \text{ERR} \notin E \}$$
- ▶ Could we postulate $\text{TR}(p) = \text{VTr}(p)$?
- ▶ No!
- ▶ $\text{VTr}(p)$ is prefix-complete but not necessarily serial!

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Non-serial program

- ▶ Program p below has non-serial set $\text{VTr}(p)$:

(define (bug)
 (sync (wait 1 (/ 1 0))))

- ▶ All traces in $\text{Tr}(p)$ below one second contain ERR, and thus are not in the set $\text{VTr}(p)$

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Fix 2

- ▶ Let $\text{Tr}(p)$ be the largest subset of $\text{VTr}(p)$ that is serial
 - Being serial is a closure property
 - Hence this subset is well-defined
- ▶ $\text{Tr}(p)$ is now serial, prefix-complete by construction
- ▶ If it is non-empty, then p is valid specification interpreted like this

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Motivation

- ▶ No trace out of $VTr(p)$ should be valid (largest subset of $ETr(p)$ not containing ERRs)
- ▶ Suppose T belongs to $VTr(p)$, but it has not arbitrarily long extensions
- ▶ Then an execution error is guaranteed after a finite time
- ▶ Hence T must be considered a phantom trace
- ▶ $Tr(p)$ is now the largest subset of $VTr(p)$ not containing these traces
- ▶ $Tr(p)$ fulfils the properties required from a set of valid traces

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Testing of Concurrent Systems 2004

Lecture 8
12th Oct 2004

Computational view

- ▶ Given a program p and a trace T , it is difficult to check if $T \in Tr(p)$, from a computation point of view
 - Checking $T \in ETr(p)$ is an unsolvable problem
 - Checking $T \in Tr(p)$ *additionally* requires checking that there exists at least one family of arbitrarily long extensions of T

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Computational view continued

- ▶ Using $Tr(p)$ as a set of valid traces causes thus some real world complications—in the general case
- ▶ But if program p e.g.
 - always accepts all inputs, and
 - never crashes,
- ▶ then $Tr(p) = ETr(p)$, and we are left “only” with the trace inclusion check

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The “require” procedure

- ▶ Assume the following definition:

```
(define (require b)
  (if (not b) (/ 1 0)))
```

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Require example

```
(define (echo)
  (sync (input x (require (integer? x))
        (wait-and-send x))))
(define (wait-and-send x)
  (sync (input y (wait-and-send x))
        (wait (/ (+ 1 (random 99)) 1000)
              (sync (output x (echo))))))
(echo)
```

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Require example continued

- ▶ Let p be the program from previous slide
- ▶ Let $T = \langle \{ \text{"hello"}_{in}, 1 \}, 3 \rangle$
- ▶ Note: $T \notin \text{Tr}(p)$
- ▶ What is $\text{verdict}(T, p)$?

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Verdict

- ▶ Verdict is `ERROR`
- ▶ The set of valid prefixes of T is $\{ \langle \emptyset, t \rangle \mid 0 \leq t \leq 1 \}$.
- ▶ Namely, "hello" causes the call to require to cause division by zero
 - Execution causes eventually `ERR` output
 - Hence the trace does not belong to the maximal serial subset
 - The input "hello" is the problem \rightarrow tester error

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Use of require

- ▶ Require is a device for "intensional" specifications
- ▶ Can be mind-boggling
- ▶ Consider the following example

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Require trick

```
(define (guess)
  (let ((v (any-integer)))
    (sync (input x
              (require (= v x)
                       (sync (wait 0.1
                               (sync (output "ok"
                                         (wait-for-ever))))))))))
  (define (wait-for-ever)
    (sync (input x (wait-for-ever))))
  (guess))
```

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Require trick (2)

- ▶ Let $T = \langle \langle 3, 0 \rangle, 10 \rangle$
- ▶ What should be $\text{verdict}(T, p)$???
- ▶ The solution is...

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Require trick conclusion

- ▶ ... FAIL.
- ▶ Can you understand why?

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Testing of Concurrent Systems 2004

Lecture 9
19th Oct 2004

Course this far

14.9	1	▶ Introduction, general concepts, traces
	2	▶ Concurrent Scheme
21.9	3	▶ Traces, specifications
	4	▶ Seriality, execution introduction
28.9	5	▶ Test steps and execution
	6	▶ Test verdicts
12.10	7	▶ Scheme programs as implementations, testing strategies, testers, and specifications
	8	

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Summary of last lecture

- ▶ Scheme programs as implementations and specifications
- ▶ ETr(p), VTr(p), Tr(p)

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Conformance?

- ▶ What does it mean that a system conforms to a specification?
 - System functions as specified
 - System passes all tests
 - Which “all” tests?
 - System passes every “test” that is “correct”
 - What is “a test”? What is “a correct test?”

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What is “a test”?

- ▶ A test = ?
 - a specific testing strategy
 - a specific test execution trace
 - a specific tester
 - a specific tester execution
- ▶ A correct test = ?
 - A test execution trace with verdict \neq ERROR
 - A testing strategy or tester that “never works illegally”
 - What does this mean?

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Correct testing strategies

- ▶ A testing strategy s is correct with respect to a specification S if for any implementation i :
 $P[i,s,T] > 0 \Rightarrow \text{verdict}(T, S) \neq \text{ERROR}$
- ▶ Denote by $\text{CT}(S)$ the set of all correct testing strategies with respect to S

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Synthetic correct strategies

- ▶ A correct testing strategy can be (informally) constructed by the following loop:
 - Guess the next action (send/wait) so that a valid trace extension will result
 - Execute the chosen action, observing the actions of the SUT
 - Restart loop
- ▶ More on this on the second half!
- ▶ Shows that correct testing strategies exist
 - Possible because of the seriality of valid set of traces
- ▶ In real life computationally intensive

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Correct strategies ctd

- ▶ If we assume these synthetic strategies belong to the set of available testing strategies...
- ▶ ... then all correct and failing behaviours can be constructed against correct testing strategies.
- ▶ Make this assumption for now.

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Trace taxonomy

- ▶ Let p, p' be Scheme programs
- ▶ For every trace T , one of the following is true:
 - There exists $s \in CT(p')$ such that $P[p, s, T] > 0$
 - There exists s , but none in $CT(p')$, such that $P[p, s, T] > 0$
 - There does not exist any s such that $P[p, s, T] > 0$

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Trace taxonomy ctd

- ▶ Furthermore, for every trace T it holds that $\text{verdict}(T, p')$ is one of PASS, FAIL, ERROR
- ▶ We ignore ambiguous specifications (\rightarrow verdict CONFUSED) for now

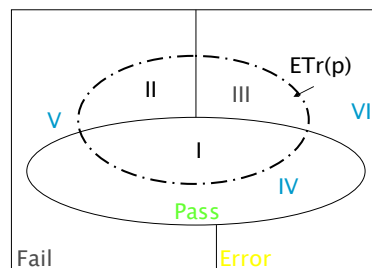
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Matrix for a trace T

Verdict \rightarrow	PASS	FAIL	ERROR
Condition			
$\exists s \in CT(p')$: $P[p, s, T] > 0$	Possible	Possible	Not possible (def. correct strategy)
$\exists s: P[p, s, T] > 0$ $\forall s: P[p, s, T] > 0 \Rightarrow s \notin CT(p')$	Not possible (synthetic testers)	Not possible (synthetic testers)	Possible
$\nexists s: P[p, s, T] > 0$	Possible	Possible	Possible

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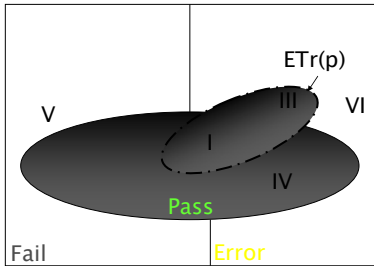
Trace map of a general system



CORRECT AND INCORRECT TESTERS
 I: correct, producible behaviour
 II: incorrect, producible
INCORRECT TESTERS ONLY
 III: producible behaviour against malfunctioning environment only
NO TESTERS AT ALL
 IV: correct behaviour not implemented
 V: incorrect behaviour not implemented
 VI: behaviour involving malfunctioning environment that has not been implemented

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Trace map of a correct system



CORRECT AND INCORRECT TESTERS

I: correct, producible behaviour

INCORRECT TESTERS ONLY

III: producible behaviour against malfunctioning environment

NO TESTERS AT ALL

IV: correct behaviour not implemented

V: incorrect behaviour not implemented

VI: behaviour involving malfunctioning environment that has not been implemented

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Correct system ctd.

- ▶ If we restrict ourselves to correct testers, then
- ▶ all behaviour that can be generated is included within the set of valid traces $Tr(S)$.

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Execution against correct strategies

- ▶ Recall

$$ETr(i) = \{ T \mid \exists s \in \mathbb{T} : P[i,s,T] > 0 \}$$
- ▶ Define now

$$ETr(i, S) = \{ T \mid \exists s \in CT(S) : P[i,s,T] > 0 \}$$
- ▶ Here $CT(S)$ is the set of testing strategies correct with respect to S
- ▶ Note $ETr(i, S) \subseteq ETr(i)$ for all S

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Continued...

- ▶ We have now eliminated the **ERROR** verdict
- ▶ Suppose for all $s \in CT(p')$,

$$P[p, s, T] > 0 \text{ implies } \text{verdict}(T, p') \neq \text{FAIL}$$
- ▶ Then (assuming unambiguous specifications),

$$P[p, s, T] > 0 \text{ implies } \text{verdict}(T, p') = \text{PASS}$$
- ▶ Hence, $ETr(p, p') \subseteq Tr(p')$

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Conclusion

- ▶ We have thus reduced the conformance of a program p to a specification p' to the equation

$$ETr(p, p') \subseteq Tr(p')$$
- ▶ This is the underlying notion of conformance in the known theory of formal conformance testing

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Conclusion ctd.

- ▶ Conformance = trace inclusion
 - Traces generated by implementation are included in those generated by specification
 - Incorrectly generated/out-of-specification traces must be excluded
 - Note: no explicit mention of single testing strategies above!

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Implications

- ▶ Quantifying over all testers leads to simple trace set inclusion
- ▶ This trace set inclusion can be also checked for directly under suitable conditions → model checking
- ▶ Thus formal conformance testing = “partial model checking”

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Note

- ▶ Note that $ETr(p, p)$ is not necessarily a subset of $Tr(p)$
- ▶ There are systems that are not conforming to themselves!

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Example

```
(let ((x (choose)))
  (if x
    (wait-for-ever)
    (begin
      (sync (output "alert"))
      (/ 1 0)
      (wait-for-ever))))
```

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Example ctd.

- ▶ This program p is not conforming to itself: $ETr(p, p)$ is not a subset of $Tr(p)$
- ▶ Can you see why?
- ▶ Systems with internal computation errors are not self-conforming
- ▶ What about invalid inputs?

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Example 2

```
(sync)
```

- ▶ This program p is self-conforming
- ▶ Every testing strategy in $CT(p)$ is silent
- ▶ Hence, the resulting traces are completely void of events
- ▶ Hence $ETr(p, p)$ contains only silent traces, which are contained in $Tr(p)$

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Example 3

```
(let ((x (receive-a-truth-value)))
  (if x
    (wait-for-ever)
    (begin
      (sync (output "alert"))
      (/ 1 0)
      (wait-for-ever))))

(define (receive-a-truth-value)
  (sync (input x (require (boolean? x) x))))
```

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Example ctd.

- ▶ This program p is conforming to itself
- ▶ The input $\#f$ (false) is not valid, because it leads unavoidably to execution error; hence traces with $\#f$ input are not in $\text{Tr}(p)$
- ▶ $\text{ETr}(p, p)$ is a subset of $\text{Tr}(p)$

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Testing of Concurrent Systems 2004

Lecture 10
19th Oct 2004

Specification-based testing algorithms

- ▶ Algorithms for running testing, based on a specification

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Basic on-the-fly algorithm

```

E := ∅, clock := 0
while [ true ]
  Xr := { <E, clock+ε> | ε > 0, <E, clock+ε> ∈ Tr(S) }
  Xin := { <E ∪ <m, clock>, clock+ε> | m ∈ Σin, ε > 0,
           <E ∪ <m, clock>, clock+ε> ∈ Tr(S) }
  Xout := { <E ∪ <m, clock>, clock+ε> | m ∈ Σout, ε > 0,
           <E ∪ <m, clock>, clock+ε> ∈ Tr(S) }
  N := Xr ∪ Xin ∪ Xout
  if [ N = ∅ ] then FAIL
  if [ stopping criterion ] then PASS
  choose T = <E', t> from N
  if T|clock ∈ Σin then { send T|clock, E := E ∪ <T|clock, clock> }
  wait for input until t // t > clock
  if [ input m received at time t' (clock ≤ t' < t) ]
    then E := E ∪ <m, t'>, clock := t'
    else clock := t
  
```

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Correctness arguments

- ▶ $\langle E, \text{clock} \rangle$ is “current” trace
- ▶ If there is no proper extension of $\langle E, \text{clock} \rangle$ in $\text{Tr}(S)$, we give FAIL verdict
 - FAIL or ERROR is correct, must show that ERROR is unnecessary
- ▶ Otherwise we “hypothesize” an extension of at most one, immediately occurring extra event
 - If the event is input to SUT, we produce that
 - The extension is legal (in $\text{Tr}(S)$)
- ▶ We wait until the end of the extension
- ▶ If SUT produces events, these are recorded
- ▶ We now claim that ERROR verdict cannot result

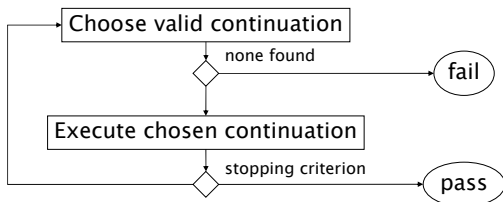
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Errors?

- ▶ Suppose the algorithm produces trace T such that $\text{verdict}(T, S) = \text{ERROR}$
- ▶ Hence $T \notin \text{Tr}(S)$. There exists time t at which T has deviated from the longest valid prefix of it
- ▶ Every proper extension of a prefix $T[t-\epsilon]$ for sufficiently small ϵ differs from T by change of input behaviour
- ▶ One of the prefixes of T is the trace at the last loop of the algorithm where the trace is still valid; the next trace differs from correct traces first by input behaviour
- ▶ But input behaviour is always chosen so that it does not lead to outside valid behaviour ($\text{Tr}(S)$)
- ▶ Hence ERROR verdict is impossible
- ▶ This is a correct tester for S , regardless of the choice structure

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Abstract version



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Choosing test steps

- ▶ How to choose a test step = how to choose next continuation = testing heuristic
- ▶ Where to focus
- ▶ Where to “lead” the system under test

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Overview

- ▶ This is a planning problem
- ▶ Assume we can somehow attach “value” to executed test runs
- ▶ Test runs that exercise “important parts” of the specification have more value
- ▶ We want to create a plan of correct test execution that results in a test run with high value
- ▶ But note that we don’t know what the SUT will do!

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Planning types

- ▶ Conformant planning = linear plan that achieves its goal, no matter what the SUT does
- ▶ Single-agent planning = co-operative planning = plan that assumes that SUT co-operates
- ▶ Adversarial planning = planning against enemy = plan that assumes that SUT actively resists testing
- ▶ Stochastic planning = planning against nature = plan that assumes that SUT makes its own choices stochastically

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Example

- ▶ Test that you can get 6 by throwing die
- ▶ Conformant plan: none, as there is no way to enforce the die to give 6
- ▶ Single-agent plan: roll once—the die will co-operate and give 6
- ▶ Adversarial plan: no plan—how many times you roll, the die will always give something else than 6
- ▶ Stochastic plan: roll the die until you get 6—the expected number of rolls is 6

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Computational aspects

- ▶ Planning in general is very difficult
- ▶ Conformant plans do not always exist
- ▶ Single-agent planning is in practice cheaper than adversarial or stochastic planning

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Discussion

- ▶ In practice SUTs are not co-operating nor adversarial; they are independent and stochastic, but their stochastic choice functions are not known
- ▶ Co-operative planning is a “quick heuristic”
- ▶ Adversarial planning is “worst case analysis” which guarantees in theory best worst-case performance—but is computationally very expensive
- ▶ Conformant planning only for simple systems

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When to stop testing?

- ▶ Two heuristic problems in testing
 - What to do
 - When to stop
- ▶ If you have arbitrarily much time, you should test arbitrarily long
- ▶ In practice there is a trade-off between better testing and spending more resources
- ▶ This is the “stopping criterion”
- ▶ Trade-offs can be analyzed using rational decision theory and like theories
 - More on this later

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Summary

- ▶ Basic on-the-fly algorithm
- ▶ Planning types
- ▶ Stopping criterion

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Testing of Concurrent Systems 2004

Lecture 11
16th Oct 2004

Course this far

14.9	1	▶ Introduction, general concepts, traces
	2	▶ Concurrent Scheme
21.9	3	▶ Traces, specifications
	4	▶ Seriality, execution introduction
28.9	5	▶ Test steps and execution
	6	▶ Test verdicts
12.10	7	▶ Scheme programs as implementations, testing strategies, testers, and specifications
	8	
19.10	9	▶ Conformance = trace inclusion
	10	▶ Basic on-the-fly testing algorithm

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Economics of testing

- ▶ We know what testing is
- ▶ We know how we can test (at least basically)
- ▶ But why we should test?

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Testing is economic activity

- ▶ Testing costs
 - Money
 - Working time
 - Other resources
- ▶ Because it costs, there must be a pay-off
- ▶ What is the pay-off from testing?

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Pay-off of testing

- ▶ Detection of bugs or faults
 - Only a known bug can be fixed
 - Knowledge of a bug is valuable
- ▶ Increased confidence
 - = reduced risk of malfunctioning
 - Can be obtained without changing the SUT!

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Rationale for testing

- ▶ We pay the cost of testing in order to reduce the risk of system malfunctioning
 - Additionally, we can spot defects, but we do not know beforehand if that will happen
- ▶ How risk reduction happens?
- ▶ How useful is it? How can we quantify it?
- ▶ We are comparing two basically incompatible things: money and risk

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Basic utility theory

- ▶ Assume you make a choice between a set of alternatives $\alpha_1, \alpha_2, \dots$
- ▶ A rational choice is to choose the alternative that is most useful = has highest utility

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Utility values

- ▶ Denote the utility of an alternative α by $u(\alpha)$; it is a real number
- ▶ Rational agent chooses alternative α_i such that $u(\alpha_i)$ is the maximum of all utilities (assume the maximum exists)

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Lotteries

- ▶ A lottery L is a probability distribution over a set of alternatives A :
$$L : A \rightarrow [0,1]$$
- ▶ The expected utility theory assumes that the utility of L is given by

$$u(L) = \sum_{\alpha \in A} L(\alpha)u(\alpha)$$

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Utility of money

- ▶ Money has in general nonlinear utility
- ▶ Compare receiving 1,000,000 € with taking part in the lottery L over 0 €, 2,000,000 € such that $L(0 \text{ €}) = \frac{1}{2}$ and $L(2,000,000 \text{ €}) = \frac{1}{2}$. Which one would you choose?
- ▶ We assume linear utility, so we can use money as unit of utility (for the sake of concreteness)

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Failure models

- ▶ Recall that a failure model is a function from specifications to probability distributions over implementations

$$\mu : \mathcal{S} \rightarrow (\mathcal{I} \rightarrow [0, 1])$$

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Transforming distributions

- ▶ Assume S is a specification and $\psi = \mu(S)$
- ▶ Unknown SUT i is chosen according to $\mu(S)$ —the a priori distribution
- ▶ We test i with strategy s , observing trace T (with verdict $PASS$). What is the a posteriori distribution of SUTs?

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Linking distributions

- ▶ Denote the a posteriori distribution by ψ'
- ▶ ψ is transformed to ψ' by s, T
- ▶ How?
- ▶ We will employ the **Bayes' Rule**

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Bayes' Rule

- ▶ The Bayes' Rule is a basic rule of conditional probability:

$$P(B|A) = P(A|B) P(B) / P(A)$$

- ▶ Derivation:

- $P(B|A) =$
- $P(A,B)/P(A) =$
- $P(A,B) P(B) / [P(B) P(A)] =$
- $P(A|B) P(B) / P(A)$

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Compute ψ'

- ▶ We have produced trace T against unknown implementation with strategy s , a priori distribution of implementations being ψ
- ▶ The a priori probability for trace T is $\sum_i \psi[i] P[i,s,T]$
- ▶ The a priori probability for implementation i is $\psi[i]$
- ▶ What is the a posteriori probability distribution for an implementation i ?

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Compute ψ' (2)

- ▶ $\psi'(\psi, s, T)[i] =$
$$P[i, s, T] \psi[i] / (\sum_{i^*} \psi(i^*) P[i^*, s, T])$$
- ▶ If $P[i, s, T] = 0$ then $\psi'[i] = 0$
- ▶ If $\psi[i] = 0$ then $\psi'[i] = 0$
- ▶ if $P[i, s, T] = kP[i', s, T]$, then $\psi'[i]/\psi[i] = k\psi'[i']/\psi[i']$

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Correct implementations

- ▶ Let $A(S)$ be the set of (absolutely) correct implementations of specification S
- ▶ The a priori probability for a correct system is $C = \sum_{i \in A(S)} \psi[i]$
- ▶ The a posteriori probability for a correct system after testing is $C' = \sum_{i \in A(S)} \psi'(\psi, s, T)[i]$
- ▶ If testing results in **PASS**, is it automatically true that $C' \geq C$..?

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Increasing correctness

- ▶ The answer is, unfortunately, no.
- ▶ It is possible that $C' < C$.
- ▶ This is a strange paradox of testing: sometimes a specific test run yields verdict **PASS**, but still decreases the probability of having a correct system.
- ▶ But we don't dwell longer on this.

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Economic implications

- ▶ Suppose an correct system has utility of X (€), incorrect system has utility 0
- ▶ The expected utility of unknown SUT is CX before testing and $C'X$ after testing
- ▶ If $C' > C$, it pays off to pay less than $(C' - C)X$ for testing

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Shortcomings

- ▶ In general we can't know C' before we have done the testing
- ▶ Also, the utility of a correct system or the disutility of an incorrect one is not constant. It depends on use, and is itself probabilistic.
- ▶ More on this on the next lecture...

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Testing of Concurrent Systems 2004

Lecture 12
26th Oct 2004

Incorrect systems introduction

- ▶ Denote by $P_n[i,s,T]$ the probability that trace T is produced against implementation i with strategy s in exactly n test steps.
- ▶ The probability that the execution after n steps is incorrect w.r.t. S is given by

$$F_n[i,s,S] = \sum_{T \notin \text{Tr}(S)} P_n[i,s,T]$$

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Properties of F_n

- ▶ $F_0[i,s,S] = 0$
- ▶ $F_n[i,s,S] \leq F_{n+1}[i,s,S]$

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Expected length of correct execution

- ▶ Suppose the series

$$E = \sum_{i>0} (i - 1)(F_i - F_{i-1})$$

converges. Then E is the expected length of a correct execution before failure.

- ▶ $F_i - F_{i-1}$ is the probability that the first failure occurs at i^{th} step.

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Alternative form

- ▶ E can be computed also as

$$\sum_{i>0} (1 - F_i)$$

- ▶ Summing up the previous series upto k , we get

$$(\sum_{0<i\leq k} -F_i) + kF_{k+1}$$

- ▶ Because the series converges, $\lim_{n \rightarrow \infty} F_n = 1$. Hence in the limit, we get

$$(\sum_{0<i\leq k} -F_i) + k = (\sum_{0<i\leq k} 1 - F_i).$$

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When E does not converge

- ▶ What if E is not converging?
- ▶ If $\lim_{n \rightarrow \infty} F_n = 0$, the system is absolutely correct with respect to the strategy s
- ▶ Otherwise (larger limit), the relative probability of system failure must decrease in time
 - If $(1 - F_n)/(1 - F_{n-1}) < \alpha < 1$ (α constant), the series converges

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E does not converge (ctd)

- ▶ Those cases where E does not converge but the system is not absolutely correct can be ruled out as “unnatural”
 - E.g. systems that are capable of failing only at “system start” but not later—what is a system start? Is reset not allowed never afterwards?
- ▶ This leaves us with two system classes w.r.t. a strategy s
 - Absolutely correct systems
 - Those with finite expected correct execution length

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Economic considerations

- ▶ Assume every use step yields benefit B, and every system failure costs F
- ▶ The discounted benefit per use step is $B - F/E$
- ▶ If $B - F/E < 0$, system is useless
- ▶ For absolutely correct system (with respect to a strategy), the discounted benefit is just B

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Use strategy

- ▶ If we fix a use strategy s , we can compute the expected benefit from a distribution ψ as usual:
$$\sum_i \psi[i](B - F/E_i)$$
- ▶ After testing, this becomes
$$\sum_i \psi'[i](B - F/E_i)$$
- ▶ But we can't still quantify the benefit of testing...

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... ctd ...

- ▶ ... because we do not know how long the system is used
- ▶ Assume life cycle of N steps
- ▶ Total increase of utility is
$$N \sum_i (\psi'[i] - \psi[i])(B - F/E_i)$$
- ▶ Bounded above by NB
- ▶ Computing in practice basically impossible—why?

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Notes

- ▶ Analysis of this kind depends heavily on the failure model μ
- ▶ How it is obtained?
- ▶ Can the calculations be carried actually out?
- ▶ What is the utility?

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Testing of Concurrent Systems 2004

Lecture 13
2nd Nov 2004

Course this far

14.9	1	▶ Introduction, general concepts, traces
	2	▶ Concurrent Scheme
21.9	3	▶ Traces, specifications
	4	▶ Seriality, execution introduction
28.9	5	▶ Test steps and execution
	6	▶ Test verdicts
12.10	7	▶ Scheme programs as implementations, testing strategies, testers, and specifications
	8	
19.10	9	▶ Conformance = trace inclusion
	10	▶ Basic on-the-fly testing algorithm
26.10	11	▶ Rational decision, "economics of testing"
	12	

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Review

- ▶ Comments from the last lecture:
 - "What is the use of this?"
- ▶ After the infamous economics of testing lecture, we move (back) to something more concrete, namely
- ▶ testing algorithms.

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Basic on-the-fly algorithm

- ▶ We review the algorithm from lecture 10 and insert a small fix
- ▶ The original algorithm is correct, but this fix makes later developments more straightforward

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Basic on-the-fly algorithm

```

E := ∅, clock := 0
while [ true ]
  Xr := { <E, clock+ε> | ε > 0, <E, clock+ε> ∈ Tr(S) }
  Xin := { <E ∪ <m, clock>, clock+ε> | m ∈ Σin, ε > 0,
            <E ∪ <m, clock>, clock+ε> ∈ Tr(S) }
  Xout := { <E ∪ <m, clock>, clock+ε> | m ∈ Σout, ε > 0,
            <E ∪ <m, clock>, clock+ε> ∈ Tr(S) }
  N := Xr ∪ Xin ∪ Xout
  if [ N = ∅ ] then FAIL
  if [ stopping criterion ] then PASS
  choose T = <E', t> from N
  if T|clock ∈ Σin then { send T|clock, E := E ∪ <T|clock, clock> }
  wait for input until t // t > clock
  if [ input m received at time t' (clock ≤ t' < t) ]
    then E := E ∪ <m, t'>, clock := t' + ε for a "very" small ε
    else clock := t
    
```

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Purpose of the fix

- ▶ Now $\langle E, \text{clock} \rangle$ is always a valid trace object
- ▶ This did not hold previously, even though the algorithm was correct
- ▶ A practical implementation can handle the issue differently

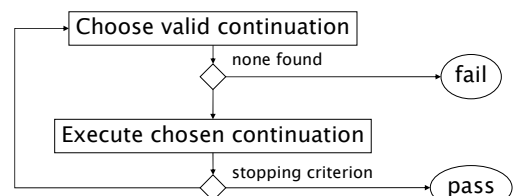
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Review continues

- ▶ We review the abstract version

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Abstract version



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A planning version

- ▶ A test execution algorithm that “aims” at a specific trace
- ▶ The trace is chosen by the algorithm, in a yet unspecified manner

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Plan-oriented testing algorithm

```

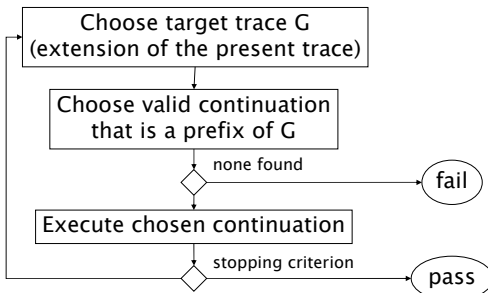
E := ∅, clock := 0
while [ true ]
  Choose a suitable G from Tr(S) s.t. <E, clock> < G
  Xr := { <E, clock+ε> | ε > 0, <E, clock+ε> < G }
  Xin := { <E ∪ <m, clock>, clock+ε> | m ∈ Σin, ε > 0,
           <E ∪ <m, clock>, clock+ε> < G }
  Xout := { <E ∪ <m, clock>, clock+ε> | m ∈ Σout, ε > 0,
           <E ∪ <m, clock>, clock+ε> < G }
  N := Xr ∪ Xin ∪ Xout
  if [ N = ∅ ] then FAIL
  if [ stopping criterion ] then PASS
  choose T = <E', t> from N
  if T|clock ∈ Σin then { send T|clock; E := E ∪ T|clock; clock > }
  wait for input until t // t > clock
  if [ input m received at time t' (clock ≤ t' < t) ]
    then E := E ∪ <m, t'>, clock := t' + ε for a “very” small ε
    else clock := t
  
```

New choice

Reduced options

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Abstract version



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Comments

- ▶ Decision about “where to proceed” has been factored into two decisions:
 - What is the aim
 - What is the next step towards the aim

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Property covering

- ▶ Assume there exists a universe of “properties”, and a procedure UniversalPropertyCheck that maps a trace and a specification to a set of properties
 - A set of properties that every “execution” of a specification (as a reference implementation) that produces the given trace has
 - We will see a concrete implementation

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Property covering (ctd.)

- ▶ Furthermore, assume there exists another procedure PlanForMoreProperties that maps a set of properties, a trace, and a specification, to a new “goal” trace, such that an execution leading to the trace covers more properties
- ▶ We get a greedy property-covering testing algorithm

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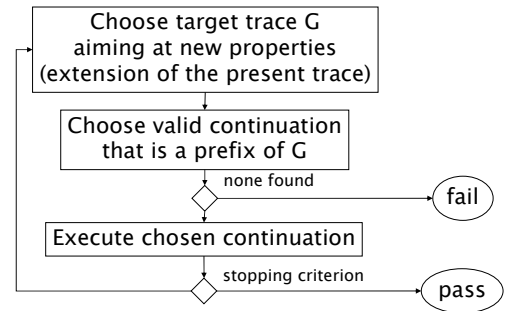
Property-covering testing algorithm

```

E := ∅, clock := 0, P := ∅
while [ true ]
  P := P ∪ UniversalPropertyCheck(<E, clock>, S)
  G := PlanForMoreProperties(P, <E, clock>, S)
  if [ no G found ]
    Choose a suitable G from Tr(S) s.t. <E, clock> < G
  Xr := { <E, clock+ε> | ε > 0, <E, clock+ε> < G }
  Xin := { <E ∪ <m, clock>, clock+ε> | m ∈ Σin, ε > 0,
           <E ∪ <m, clock>, clock+ε> < G }
  Xout := { <E ∪ <m, clock>, clock+ε> | m ∈ Σout, ε > 0,
           <E ∪ <m, clock>, clock+ε> < G }
  N := Xr ∪ Xin ∪ Xout
  if [ N = ∅ ] then FAIL
  if [ stopping criterion ] then PASS
  choose T = <E', t'> from N
  if Tclock ∈ Σin then { send Tclock, E := E ∪ <Tclock, clock> }
  wait for input until t' // t' > clock
  if [ input m received at time t' (clock ≤ t' < t) ]
    then E := E ∪ <m, t'>, clock := t' + ε for a "very" small ε
    else clock := t
  
```

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Abstract version



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Testing of Concurrent Systems 2004

Lecture 13
2nd Nov 2004

A dive deeper

- ▶ How do we check if $T \in \text{Tr}(S)$?
- ▶ How do we compute the "properties" that a trace "necessarily" covers?
- ▶ How do we compute goal traces?

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State space based computation

- ▶ The execution function ξ gives external behaviour, but it thus abstracts away the "internals" of a specification
- ▶ This is not practical from the computation point of view
- ▶ Typically also the internal and "silent" computation steps count and cause difficulties
- ▶ → internal state spaces

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State spaces

- ▶ A state is (here) a pair $\langle c, T \rangle$ where c is an "internal control state" and T is an I/O trace produced "until now"
- ▶ For every state s , there exists a set of successor states (potentially infinite), denoted by $\text{next}(s)$
- ▶ If $s' \in \text{next}(s)$, we write also $s \rightarrow s'$

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State spaces

- ▶ Assume we can associate with a specification
 - an initial state $s_0 = \langle c_0, \langle \emptyset, 0 \rangle \rangle$
 - next state relation
- ▶ $\text{Tr}(S) = \{ T \mid \exists \langle c, T \rangle : s_0 \rightarrow^* \langle c, T \rangle \}$
- ▶ We assume that the seriality requirement is fulfilled implicitly in the state space
 - But this is not necessarily the case in reality

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Basic trace inclusion check algorithm

```
W := {s0}
V := ∅
While W ≠ ∅
  Choose <c,T> from W
  If T = T*
    Return FOUND
  Else if T < T*
    W := W - {<c,T>}
    V := V ∪ {<c,T>}
    W := W ∪ (next(<c,T>) - V)
Return NOT FOUND
```

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Comments

- ▶ If next(s) is infinite, won't work
 - Symbolic methods needed
- ▶ Does not necessarily terminate if
 - Infinite branches (next(s) infinite)
 - Arbitrarily many computation steps possible in finite real time (unboundedly many steps possible before trace end time stamp reaches a constant t)

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Properties

- ▶ Suppose we can attach a set of properties P to every transition from s to s'
- ▶ Write $s \rightarrow_P s'$ if there is a transition from s to s' with properties P

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UniversalPropertyCheck(T*,S)

```
W := {<s0, ∅>}
V := ∅
P := everything
While W ≠ ∅
  Choose <<c,T>,π> from W
  If T = T*
    P := P ∩ π
  Else if T < T*
    W := W - {<<c,T>,π>}
    V := V ∪ {<<c,T>, π>}
    N := { <s', π'> | s →_Q s', π' = π ∪ Q }
    W := W ∪ (N - V)
If P is everything
  Return Trace not found
Else
  Return P
```

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Comments

- ▶ Computes the set of properties that every execution that produces a given trace must have

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PlanForMoreProperties(P,T*,S)

```
W := {<s0, ∅>}
V := ∅
While W≠∅
  Choose <<c,T>,π> from W
  If T ≪ T* or T* ≪ T
    If π ≰ P and T* < T
      If (UniversalPropertyCheck(T,S) ≰ P)
        Return T
  Else
    W := W - {<<c,T>,π>}
    V := V ∪ {<<c,T>, π>}
    N := { <s', π'> | s →Q s', π' = π ∪ Q }
    W := W ∪ (N - V)
Return Trace not found
```

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Comments

- ▶ Finds a trace that implies properties that are not present in the set P
- ▶ Before the UniversalPropertyCheck, it holds that at least one way to reach the trace T implies new properties
- ▶ The UniversalPropertyCheck call is used to ensure that this holds for all alternative executions as well

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Discussion

- ▶ Property = interesting feature in specification
- ▶ For example, a property = a state in a state chart model, or a Scheme expression in a Scheme reference implementation
- ▶ Intuition: it is good to exercise “many parts” of reference implementation rather than “few parts”
- ▶ But...

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Discussion (ctd)

- ▶ ... as mentioned on the “economics” lecture, it is impossible to prove that this would be a good thing
- ▶ So just a heuristic

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Properties = coverage measures

- ▶ Known or used ways to measure “coverage” (properties)
 - Transitions of a state chart
 - States of a state chart
 - Lines visited
 - Branch coverage (true and false branches of switches)
 - Condition coverage (true and false valuations of “atomic” subexpressions in switch expressions)
 - ...

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Improvements

- ▶ Greedy algorithms are not usually optimal → a better planner could reach all interesting properties in less testing steps
 - However becomes computationally more intensive
 - Greedy algorithm works rather well in practice

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Symbolic execution

- ▶ If next(s) sets are infinite, the algorithms can't be realized "as such"
- ▶ Symbolic execution is needed
 - An algorithmic solution to the problem of infinite state sets
 - Well known in general
- ▶ For illustration, let us consider the trace inclusion check algorithm

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Symbolic trace inclusion check algorithm

```

W := {α[s0]}
V := ∅
While W ≠ ∅
  Choose s from W
  If NotEmpty(s ∩ LiftTrace(T*))
    Return FOUND
  Else
    W := W - {s}
    V := V ∪ {s}
    N := SymbolicSuccessors(s) ∩ LiftPrefix(T*)
    W := W ∪ (N - V)
Return NOT FOUND
    
```

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Comments

- ▶ α maps a concrete state to a symbolic state representing the singleton set consisting of the concrete state
- ▶ \sqcap computes symbolic intersection
- ▶ LiftPrefix(T*) returns a symbolic state that represents every state whose trace is either a prefix of T*, or an extension of T*
 - Replaces the check $T < T^*$
- ▶ LiftTrace(T*) returns a symbolic state that represents every state whose traces is exactly T*
 - Replaces equivalence check
- ▶ NotEmpty checks for non-empty symbolic state

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Symbolic states

- ▶ How symbolic states can be implemented?
- ▶ Many techniques known, e.g.
 - BDDs (binary decision diagrams)
 - Constraint systems
 - Linear constraints over reals (\rightarrow timed automata)
 - General constraints

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Testing of Concurrent Systems 2004

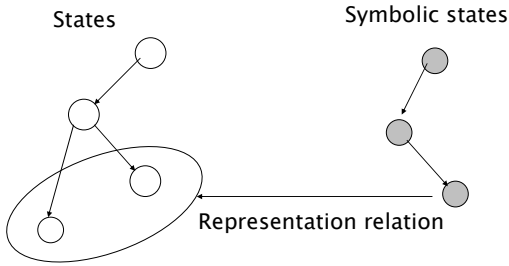
Lecture 15
15th Nov 2004

Course this far

14.9	1	▶ Introduction, general concepts, traces
	2	▶ Concurrent Scheme
21.9	3	▶ Traces, specifications
	4	▶ Seriality, execution introduction
28.9	5	▶ Test steps and execution
	6	▶ Test verdicts
12.10	7	▶ Scheme programs as implementations, testing strategies, testers, and specifications
	8	
19.10	9	▶ Conformance = trace inclusion
	10	▶ Basic on-the-fly testing algorithm
26.10	11	▶ Rational decision, "economics of testing"
	12	
2.11	13	▶ Testing algorithms, planning, property covering
	14	▶ State-space based algorithms

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Symbolic states



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Representation

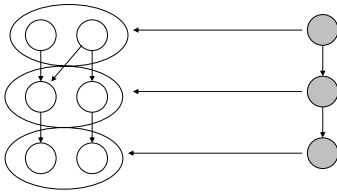
- ▶ Let z be a symbolic state
- ▶ $\gamma(z)$ is a set of states: the set of states represented by z
- ▶ For a concrete state s , $\alpha(s)$ is a symbolic state such that $\gamma(\alpha(s)) = \{s\}$

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Axiom

- ▶ If $z \rightarrow z'$, then

$$\gamma(z') = \{s' \mid \exists s \in \gamma(z): s \rightarrow s'\}$$



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Operations for symbolic states

- ▶ Emptiness check

$$\text{Empty}(z) : \gamma(z) = \emptyset$$
- ▶ Intersection

$$\gamma(z \sqcap z') = \gamma(z) \cap \gamma(z')$$
- ▶ Subsumption relation

$$z \sqsubseteq z' \Rightarrow \gamma(z) \subseteq \gamma(z')$$

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Symbolic successors

- ▶ $\text{Next}(z) = \{z' \mid z \rightarrow z'\}$
- ▶ Axiom 2:

$$s \in \gamma(z), s \rightarrow s' \text{ implies } \exists z' \in \text{Next}(z) : s' \in \gamma(z')$$

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Operations needed for symbolic trace inclusion check

- ▶ LiftTrace(T)
 - Returns z such that

$$\gamma(z) = \{s \mid \exists c : s = \langle c, T \rangle\}$$
- ▶ LiftPrefix(T)
 - Returns z such that

$$\gamma(z) = \{s \mid \exists c, T' : s = \langle c, T' \rangle, T' \preceq T\}$$

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Symbolic trace inclusion check algorithm

```

W := {α{s0}}
V := ∅
While W ≠ ∅
  Choose z from W
  If not Empty(z ∩ LiftTrace(T*))
    Return FOUND
  Else
    W := W - {z}
    V := V ∪ {z}
    N := { z' | z' ∈ Next(z), z' = z' ∩ LiftPrefix(T*) }
    W := W ∪ (N - V)
Return NOT FOUND
    
```

W = {{s₀}}

Set z contains <C, T> for some c?

Compute successors but filter out states whose traces are not prefixes of T*

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Correctness discussion

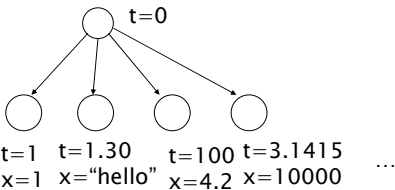
- ▶ Suppose $\gamma(z)$ are all reachable in the concrete state space
- ▶ Suppose $z \rightarrow z'$
- ▶ Then also $\gamma(z')$ are all reachable by definition
- ▶ On the other hand, suppose s is reachable, and z is reachable such that $\gamma(z)$ contains s
- ▶ Suppose $s \rightarrow s'$
- ▶ Then z' exists in the set $\text{Next}(z)$ such that $s' \in \gamma(z')$

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Symbolic states: example

```

(sync (input x
  (sync (wait 0.1
    (sync (output (+ x 1) (halt)))))))
    
```



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Discussion

- ▶ The symbolic state space depicts the whole infinite state space, but is in the example finite
- ▶ Individual states are represented symbolically as individual solutions to a constraint set

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Constraint solutions

- ▶ Constraint set: $\{X1 > 0, X3 = X1 + 0.1, \text{number } X2, X4 = X2 + 1\}$
- ▶ $X1 = 0.2, X3 = 0.3, X2 = 9, X4 = 10$ is a solution
- ▶ Corresponds to a real execution
- ▶ $X1 = -1$ does not lead to a solution
 - Negative time stamp!
- ▶ $X1 = 1, X3 = 10$ does not lead to a solution
 - Wrong wait time!
- ▶ $X2 = \text{"hello"}$ does not lead to a solution
 - Received value not number!

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Computational point of view

- ▶ Constraint sets are easy to create, difficult to solve
- ▶ Unsolvable problems abound
- ▶ But many realistic cases can be handled

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More details

- ▶ System state structure $\langle c, T \rangle$
- ▶ Assume that $\langle c, T \rangle$ is otherwise concrete represented, but that c and T can mention constraint variables
- ▶ Add a constraint set
- ▶ Symbolic state is of the form $\langle \langle c, T \rangle, C \rangle$ where C is a constraint set
- ▶ Constraint set constrains the values of the constraint variables
- ▶ A concrete state is represented iff it is obtained by replacing the constraint variables with a solution of the constraint set

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Example

- ▶ $c = [t \rightarrow X1, x \rightarrow X2, \dots]$
- ▶ $T = \langle \{ \langle X2_{in}, X1 \rangle, \langle X4_{out}, X3 \rangle \}, X3 \rangle$
- ▶ $C = \{ X1 > 0, X3 = X1 + 0.1, \text{number } X2, X4 = X2 + 1 \}$
- ▶ $\langle \langle c, T \rangle, C \rangle$ is a symbolic state

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Intersections

- ▶ We assume the symbolic states are structured so that if z and z' represent at least one concrete same state, there is 1-1 correspondence between constraint variables of the symbolic states
- ▶ This can be provided

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Intersections ctd

- ▶ We can then take two symbolic states $z = \langle \langle c, T \rangle, C \rangle$ and $z' = \langle \langle c', T' \rangle, C' \rangle$ and proceed to compute their intersection
- ▶ Map all constraint variables of z' to those of z , with mapping Q (if not possible, intersection empty)
- ▶ Intersection is $\langle \langle c, T \rangle, C \wedge Q(C') \rangle$
- ▶ Assumes constraint sets are closed under conjunction

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Intersections ctd

- ▶ To make LiftTrace, LiftPrefix work, we must also allow for a case where the control part is undefined
- ▶ $\langle \langle c, T \rangle, C \rangle \sqcap \langle \langle ?, T' \rangle, C' \rangle$:
match T against T' , then yield
 $\langle \langle c, T \rangle, C \wedge Q(C') \rangle$
- ▶ (or empty symbolic state)

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Emptiness check

- ▶ Emptiness check can be now reduced to checking for the solvability of a constraint set

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Subsumption check

- ▶ Subsumption check can be reduced now to checking that a constraint set implies another one
- ▶ To check for $C \Rightarrow C'$, check for the solvability of $C \wedge \neg C'$
- ▶ Assumes now that constraint sets are closed also under negation \rightarrow full Boolean closure

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More algorithms

- ▶ The symbolic versions of the full testing algorithms are left as an exercise for the student

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Testing of Concurrent Systems 2004

Lecture 16
15th Nov 2004

Symbolic execution of Scheme

- ▶ Let's have a simplified look on the stack-based execution of Scheme

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Scheme execution

Value stack

Term stack `(let ((x (+ 1 2))) (sync (output x)))`

Environment

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Scheme execution

Value stack

Term stack `(+ 1 2) (BIND x) (sync (output x)) (RESTORE)`

Environment

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Scheme execution

Value stack

Term stack + 1 2 (APP 2) (BIND x) (sync (output x)) (RESTOR

Environment

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Scheme execution

Value stack primitive +

Term stack 1 2 (APP 2) (BIND x) (sync (output x)) (RESTO

Environment

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Scheme execution

Value stack integer 1 primitive +

Term stack 2 (APP 2) (BIND x) (sync (output x)) (RESTOR

Environment

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Scheme execution

Value stack integer 2 integer 1 primitive +

Term stack (APP 2) (BIND x) (sync (output x)) (RESTORE)

Environment

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Scheme execution

Value stack integer 3

Term stack (BIND x) (sync (output x)) (RESTORE)

Environment

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Scheme execution

Value stack

Term stack (sync (output x)) (RESTORE)

Environment x := integer 3

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Scheme execution

Value stack `#f`

Term stack `(RESTORE)`

Environment `x := integer 3`

Events `<3, 0>`

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Scheme execution

Value stack `#f`

Term stack

Environment

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How does this work symbolically?

- ▶ Straightforward Scheme executor manipulates concrete datums (integers, booleans, ...)
- ▶ Symbolic Scheme executor manipulates constraint variables as datums

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Scheme execution

Value stack

Term stack `(sync (input x (if (> x 5) (gorble x))))`

Environment

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Scheme execution

Value stack

Term stack `(if (> x 5) (gorble x) (RESTORE))`

Environment `x := X1`

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Scheme execution

Value stack

Term stack `(> x 5) (TEST (gorble x) #void) (RESTORE)`

Environment `x := X1`

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Scheme execution

Value stack

Term stack `> x 5 (APP 2) (TEST (gorble x) #void) (RESTORE)`

Environment `x := X1`

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Scheme execution

Value stack `Primitive >`

Term stack `x 5 (APP 2) (TEST (gorble x) #void) (RESTORE)`

Environment `x := X1`

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Scheme execution

Value stack `Variable X1` `Primitive >`

Term stack `5 (APP 2) (TEST (gorble x) #void) (RESTORE)`

Environment `x := X1`

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Scheme execution

Value stack `integer 5` `variable X1` `primitive >`

Term stack `(APP 2) (TEST (gorble x) #void) (RESTORE)`

Environment `x := X1`

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Scheme execution

Value stack `variable X2`

Term stack `(TEST (gorble x) #void) (RESTORE)`

Environment `x := X1`

Constraints `X2 ⇔ (X1 > 5), number X1`

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Scheme execution

Value stack

Term stack `(gorble x) (RESTORE)`

Environment `x := X1`

Constraints `X2 ⇔ (X1 > 5), number X1, X2 = #t`

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Discussion

- ▶ Symbolic Scheme execution is a concrete instance of the symbolic state space exploration idea
- ▶ Can be used to implement formal conformance testing

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Where constraint variables come from?

- ▶ There are two causes for constraint variables in symbolic execution:
 - Internal choices (e.g. (random))
 - Input from environment (message, timeout)
- ▶ But these two cases are completely different!
 - Internal choices and input from environment correspond to decisions made by distinct parties (SUT, Tester)
 - A problem lurks...

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Alternating quantifiers!

- ▶ Basically, we would like to create testing plans that cover all potential internal choices of a correctly working SUT
- ▶ This yields to constraint solving over alternating quantifiers (→ adversarial planning)
- ▶ Seems to be computationally infeasible
- ▶ Must straighten some curves, and assume a co-operative SUT
- ▶ With a co-operative SUT, SUT choices and Tester choices are on par

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Testing of Concurrent Systems 2004

Lecture 17
23rd Nov 2004

Course this far

14.9	1	▶ Introduction, general concepts, traces	2.11	13	▶ Testing algorithms, planning, property covering
	2	▶ Concurrent Scheme		14	▶ State-space based algorithms
21.9	3	▶ Traces, specifications	16.11	15	▶ Symbolic state space exploration
	4	▶ Seriality, execution introduction		16	▶ Symbolic execution of Scheme
28.9	5	▶ Test steps and execution			
	6	▶ Test verdicts			
12.10	7	▶ Scheme programs as implementations, testing strategies, testers, and specifications			
	8				
19.10	9	▶ Conformance = trace inclusion			
	10	▶ Basic on-the-fly testing algorithm			
26.10	11	▶ Rational decision, "economics of testing"			
	12				

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Topics today

- ▶ The classic IOCO theory
- ▶ Critique of IOCO

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loco theory

- ▶ The “classic theory”
- ▶ Often referred to as the “ioco” testing theory and is quite well known among the academic peoples
- ▶ A framework developed by Tretmans, Heerink et al.
- ▶ Dates to early 90’s

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loco theory overview

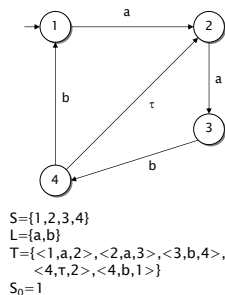
- ▶ LTSs (labeled transition systems) = finite state machines
- ▶ No notion or only a very weak notion of time
- ▶ Some tools have been developed based on the theory, for example TorX

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Labeled transition systems

- ▶ A labeled transition system is a tuple $\langle S, L, T, s_0 \rangle$ where

- S is the set of states
- L the set of transition labels
- $T \subseteq S \times L_{\tau} \times S$ the transition relation (with $L_{\tau} = L \cup \{\tau\}$)
- $s_0 \in S$ the initial state.

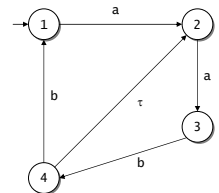


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Traces

- ▶ The traces of an LTS are obtained by “walking” in it starting from the initial state, and collecting all symbols except τ ’s which denote “silent activity” and which are removed.

ϵ a aa
 aab aabb aaba
 aabba aabab aababa



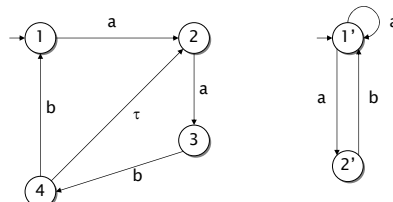
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Parallel composition

- ▶ The parallel composition of two LTSs is traditionally denoted by $L \parallel L'$.
- ▶ This construct creates a new LTS from two LTSs.
- ▶ Two LTSs run in synchrony, always taking arcs together with same labels. An exception is the τ -label which is not synchronized.
- ▶ This synchronization is not directional but completely symmetric.
 - Can be therefore called a “handshake”.

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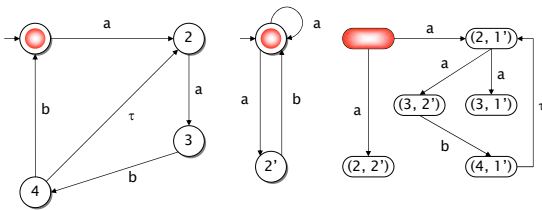
Example



- ▶ There are eight state pairs in total. So the parallel composition will have eight or less states. It is so small that we can construct it explicitly.

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Example



- ▶ The resulting LTS has only six states. The reason is that the states $\langle 1, 2' \rangle$ and $\langle 4, 2' \rangle$ are not reachable.
- ▶ The second LTS does not allow for two b's in a row.

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More on the parallel composition

- ▶ Parallel composition models “synchronous, symmetric communication” or “symmetric handshake”.
- ▶ Powerful construct: the reachability problem (= can a given composite state be reached) for parallel composed LTSs is PSPACE-complete (on the number of composed LTSs). This means that the problem is very hard.
- ▶ In the ioco testing theory, parallel composition is used to model the communication between Tester and the SUT (both are assumed to be LTSs).

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Parallel composition and realistic I/O

- ▶ In parallel composition, the two LTSs can take step with label a ($\neq \tau$) only if they do that together.
- ▶ This means that if a models, say, a message from Tester to SUT, then the SUT can refuse to receive the message (just by not having an outgoing transition with the label a).
- ▶ This is disturbing, because after all it is in the Tester's discretion to decide when to send messages and when not.
- ▶ These aspects lead us to the concept of an IOTS.

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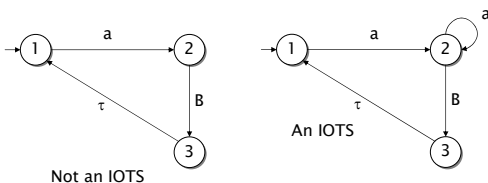
IOTS

- ▶ IOTS = Input Output Transition System.
- ▶ The set of labels L is partitioned into input labels L_I and output labels L_O .
- ▶ An IOTS is a standard LTS that has the following extra property:
- ▶ For every reachable state s in the LTS, there exists a path from s that accepts any arbitrary input label first. This means that you cannot refuse an input and that you can't deadlock.

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Example

- ▶ Assume the set of input labels is $\{a\}$ and the set of output labels is $\{B\}$.



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Testing Theory for IOTSs

- ▶ In the “ioco” testing theory, the Tester and the SUT are assumed to be IOTSs.
- ▶ Obviously, the Tester and SUT are mirror images of each other in the sense that outputs from SUT are inputs to Tester and vice versa.
- ▶ Hence, if L_O is the set of outputs from SUT, then this is the set of inputs to Tester, which must be always enabled in Tester.
- ▶ The operational specification is also an IOTS. (Actually, it can be a non-IOTS LTS—the theory speaks of “partial specifications”).

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The core idea

- ▶ Assume we have some definition of “observations” that an LTS produces; we denote this for now by $\text{obs}(L)$ for an LTS L .
- ▶ Given a tester t , SUT i and specification s , let us say that t confirms i w.r.t. s if

$$\text{obs}(t \parallel i) \subseteq \text{obs}(t \parallel s).$$
 (All the three entities are IOTSs).
- ▶ We can now say that an implementation i conforms to a specification s if all possible testers confirm i w.r.t. s .
- ▶ What are the observations?

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Basic Observations

- ▶ We assume that the observations that we can make of an LTS L are the following:
 - The set of all traces of L , plus
 - the set of those traces of L after which L can be in a deadlock
- ▶ Now write $\text{obs}(L) \subseteq \text{obs}(L')$ if the subset relation holds for both the sets mentioned above.
- ▶ This leads to the input–output testing relation \leq_{iot} . We write $i \leq_{\text{iot}} s$ to denote that i conforms to s in this sense.

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Input–output testing relation

- ▶ When an implementation conforms to a specification in the sense of \leq_{iot} ...
 - If you can produce a trace against the implementation, then you could produce the same trace against the specification (= reference implementation) (but not necessarily vice versa).
 - If you can bring the implementation into a state where it just waits for input, then you could do the same with the specification (but not necessarily vice versa).

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Alternative formulation

- ▶ An alternative way to define the same result is given next.
- ▶ $i \leq_{\text{iot}} s$ iff

$$\text{traces}(i) \subseteq \text{traces}(s) \text{ and } \text{Qtraces}(i) \subseteq \text{Qtraces}(s)$$
 where $\text{Qtraces}(L)$ is the set of those traces of L after which L can be in a state where only transitions labeled by inputs are possible (i.e. L is waiting for input and cannot proceed without one; a “quiescent state”—hence ‘Qtraces’).
- ▶ So, we see here a standard trace inclusion problem... at least almost. Note that Tester is not mentioned!

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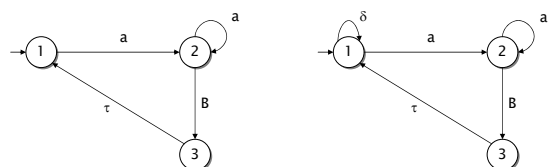
Quiescence...

- ▶ Quiescence traces model the assumption that we can detect when the SUT is not going to anything observable before it gets more input.
- ▶ Ultimately, this complication comes from the fact that there is no time in the theory.
- ▶ But actually there exists a stronger variant of this idea.

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Repetitive Quiescence

- ▶ Let us assume that we patch the SUT so that whenever it is just waiting for input, it can send out a meta-message δ which denotes “I’m waiting for input” or “I’m quiescent”.



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Repetitive Quiescence (ctd)

- ▶ The name for δ is “suspension”.
- ▶ We call the traces of an IOTS with this extension (can produce δ when no output is possible) “suspension traces”, denoted by $\text{Straces}(L)$.

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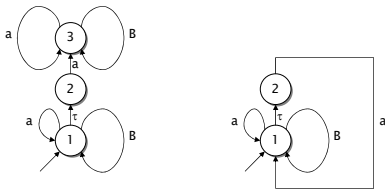
ioco relation

- ▶ Now an implementation i conforms to a specification s iff $\text{Straces}(i) \subseteq \text{Straces}(s)$.
- ▶ This corresponds to the inclusion of observations by all testers who can observe I/O behavior, deadlocks and δ s.
- ▶ This is the ioco testing relation.

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What is the Difference?

- ▶ \leq_{ipt} is based on the possibility of detecting lack of output after a test run, but only at the end of a test run.
- ▶ In ioco it is possible to detect quiescence also in the midst of a test run.



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Testing of Concurrent Systems 2004

Lecture 18
23rd Nov 2004

General comments

- ▶ ioco theory is low-level theory
 - Pragmatic systems are not given as LTSs but as Java programs, UML state charts, ...
 - Not a problem but a statement about the focus of the theory
- ▶ In principle no need to assume finite LTSes
 - But in the practice, algorithms focus on finite LTSes

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Finite LTSes

- ▶ Usually finite LTSes are assumed in the context of ioco
- ▶ But realistic systems usually have infinite or very big state graphs
- ▶ Leads to the need to do manual abstraction

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Manual abstraction in testing

- ▶ How to create a small finite state machine (i.e. LTS) from a specification generating a big/infinite state space?
- ▶ Drop out details
- ▶ Replace data with abstract placeholders

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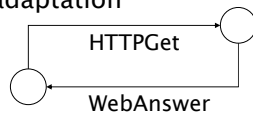
Benefits

- ▶ Resulting small state machines are easy to manipulate algorithmically
 - All kinds of interesting analyses and constructs are possible
- ▶ Strengthened focus on abstract control structure

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Cons

- ▶ Driving real testing with abstract inputs can be impossible or very difficult—the system under test wants concrete input
 - Complicated extra adaptation component



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Timing?

- ▶ The ioco theory has a weak notion of time: quiescence
- ▶ Quiescence corresponds to an abstract timeout
- ▶ However, there are no “quiescences of different length”
- ▶ Time is handled abstractly

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Adding time

- ▶ We could extend the input and output alphabets to include time stamps (as the events in our general framework)
- ▶ Then, however, both tester and SUT LTSes must become infinitely large and acyclic
- ▶ In fact, we would have some form of an alternative representation of our trace sets

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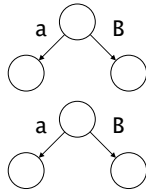
Adding time (ctd)

- ▶ But problems remain
 - Seriality
 - Progressivity
 - More definitions would be needed
- ▶ The notion of quiescence would become redundant
 - It is impossible to detect an “infinitely long” quiescence in a practical testing setup
- ▶ Tretmans et al have been working on a timed extension

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Counter-intuitive synchronization

- ▶ Consider the tester and the SUT on the right (a is input, B output)
- ▶ How do you interpret this intuitively?
- ▶ How tester and SUT negotiate the direction?
- ▶ What is the corresponding Scheme program?



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(ctd)

```
(let ((timeout (random)))
  (sync (input x (require (equal? x 'a)
    ...))
    (wait timeout
      (sync (output 'B ...))))))
```

- ▶ What is the difference? Is there any?

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Relevance of ioco theory

- ▶ A common framework
 - Many articles written
- ▶ Main contributions
 - Link the general practice of conformance testing (from telecom domain) with formal methods
 - Establish the flourishing study of formal models based conformance testing

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Conclusions

- ▶ ioco is untimed, low-level theory based on LTSes
- ▶ Practical algorithms assume finite LTSes, which leads to the problem of abstraction

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Testing of Concurrent Systems 2004

Lecture 19
30th Nov 2004

Course this far

14.9	1	▶ Introduction, general concepts, traces	2.11	13	▶ Testing algorithms, planning, property covering
	2	▶ Concurrent Scheme		14	▶ State-space based algorithms
21.9	3	▶ Traces, specifications	16.11	15	▶ Symbolic state space exploration
	4	▶ Seriality, execution introduction		16	▶ Symbolic execution of Scheme
28.9	5	▶ Test steps and execution	23.11	17	▶ ioco theory
	6	▶ Test verdicts		18	▶ Critique of ioco
12.10	7	▶ Scheme programs as implementations, testing strategies, testers, and specifications			
	8				
19.10	9	▶ Conformance = trace inclusion			
	10	▶ Basic on-the-fly testing algorithm			
26.10	11	▶ Rational decision, "economics of testing"			
	12				

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Today: “advanced” topics

- ▶ Test script generation
- ▶ Combinatorial test case design
- ▶ TTCN-3
- ▶ FCT and software process
- ▶ Implementing a toy FCT tool

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Test scripts

- ▶ Test script = explicitly given tester
- ▶ Usually assume reasonably efficient and executable implementation
- ▶ Our on-the-fly testing algorithm can be very slow
 - Planning, trace inclusion check, property coverage analysis take time

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Solution

- ▶ Try to generate an explicit-form, fast test script and use it instead of the generic algorithm
 - Do trace inclusion checks for correctness afterwards offline
 - Compute plans and property coverage offline before execution
 - In general, this is partial evaluation

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Partial evaluation

- ▶ Suppose we have a function definition

(define (f x y) ...)

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Partial evaluation (ctd)

- ▶ Let V be a certain value. Partial evaluation of f with respect to $y := V$ produces a residual procedure g on on parameter x such that

$$(g\ x) = (f\ x\ V)$$

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Partial evaluation (ctd)

- ▶ Thus, g is a specialization of f with respect to $y := V$. Similarly, f is a parameterized version of g , y being the “new” parameter.
- ▶ Partial evaluation is a well-known, advanced compiler technique.

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The link

- ▶ The on-the-fly testing algorithm is parameterized by two series of data: algorithm's internal choices (e.g. choose a valid trace T), and SUT's choices that manifest as external behaviour (e.g. receiving a message from the SUT).

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The link (ctd)

- ▶ In principle, the algorithm could have the signature
(define (otf-test internal-choices sut-behaviour) ...)
- ▶ Now partial evaluate internal-choices out

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Residual tester

- ▶ A residual tester would have the signature
(define (a-test-script sut-behaviour) ...)

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Practical notes

- ▶ There is a general way to do partial evaluation:
(define (g x)
 (let ((y V))
 <body of f>))
- ▶ But this is not interesting to us, because there is no categorical speed up
- ▶ The real problem is how to reap execution speed benefits from specialization

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Combinatorial test case design

- ▶ Basic problem
 - A system has three parameters a..c
 - Every parameter has three potential values 1..3
 - We want to test the system's behaviour with different combinations of these parameters
 - There are $3^3 = 27$ full combinations

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Pairing

- ▶ A common idea is not to test all the combinations, but to test a set of combinations such that every pair of any two values on two parameters has been tested

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Example

Run	a	b	c
#1	1	1	1
#2	1	2	2
#3	1	3	3
#4	2	1	3
#5	2	2	1
#6	2	3	2
#7	3	1	2
#8	3	2	3
#9	3	3	1

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General construction

- ▶ The idea can be generalized
- ▶ Define an arbitrary structure of partial parameter valuations to be covered
 - Explicit definition (enumerate the desired structures)
 - Implicit definition (use a language to define the structures)

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Set cover

- ▶ In an explicit form, this is the set cover problem:
- ▶ Given a set X and a set Q of subsets of X, find a minimal/small subset S of Q such that

$$\cup S = X$$

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Set cover example

- ▶ $X = \{ \text{"a=1, b=1"}, \text{"a=2, b=1"}, \dots, \text{"b=1, c=1"}, \text{"b=2, c=1"}, \dots \}$
 - Every element of X is a pair to be covered
- ▶ $Q = \{ \{ \text{"a=1, b=1"}, \text{"b=1, c=1"}, \text{"a=1, c=1"} \}, \{ \text{"a=2, b=1"}, \text{"b=1, c=1"}, \text{"a=2, c=1"} \}, \dots \}$
 - Every element of Q corresponds to a full valuation of the parameters a..c, and enumerates those pairs that the corresponding valuation covers

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Set cover ctd

- ▶ Set cover is a NP-complete problem
- ▶ An approximation algorithm exists, but not very efficient
- ▶ In practice, more direct approaches can be used which avoid the explicit enumeration of the structures
- ▶ An instance of combinatorial design

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Use within FCT

- ▶ E.g. parameter pair values can be used as properties to be covered
- ▶ An efficient property covering targeting on-the-fly testing algorithm would need to solve problems of this kind
- ▶ In practice can be also made a visible part of test design → classification tree method

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TTCN-3

- ▶ “Testing and test control notation”
- ▶ Test programming language for telco systems standardized by ETSI
 - Also used in automotive industry and related segments today
- ▶ Original focus on protocols
 - Timers
 - Concurrency
 - Data template matching

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TTCN-3 (ctd)

- ▶ Link to this class: conformance testing (in the telco way), testing of concurrent systems
- ▶ Formal conformance testing and TTCN-3 are not linked
- ▶ However, in theory test scripts generated from specifications can be rendered as TTCN-3 source code

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Testing of Concurrent Systems 2004

Lecture 20
30th Nov 2004

Formal conformance testing and software process

- ▶ How can formal conformance testing be integrated into a software process?
- ▶ Main challenges
 - Where get executable/formal specification or design?
 - Where to get a tool?
 - What kind of process support is needed?

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Specification?

- ▶ Clearly, a formal specification does not need to be in greek
- ▶ But it must have well-defined meaning
- ▶ In our context, it should be an executable reference design (e.g. in Scheme)
- ▶ Where to get it?

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How to get a reference implementation?

- ▶ First do reference implementation, then implement the real system using it as a guide
- ▶ Reverse-engineer from the implementation afterwards
- ▶ Develop at the same time as the real implementation, based on same system requirements
- ▶ Create reference implementation / system model, code-generate real system from it (→ model driven architecture)

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Tool support?

- ▶ Only emerging
- ▶ Main challenges
 - Algorithmic complexity
 - Conceptual difficulty
 - Usability
 - Business case

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Process support

- ▶ Specifications (executable reference implementations) are software artifacts!
 - They need a software process themselves
 - Testing!
 - Validation!

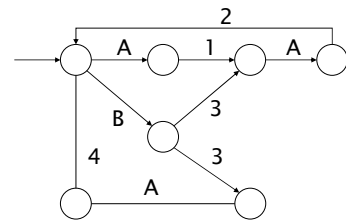
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Implementing a toy FCT tool

- ▶ Assume all I/O with system is untimed and has the form of a single stimulus + single response
- ▶ Inputs A, B, C, ..., outputs 1, 2, 3, ...
- ▶ Can draw as a state machine

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Example



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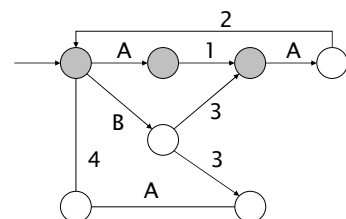
Step 1

- ▶ Create a trace inclusion checker
 - Trace e.g. "A1B3C4"
 - Return "pass" if trace found from state chart
 - Return "fail" if trace not in state chart, but every attempt to produce the trace from the state chart fails at a number (output)
 - Return "error" if trace not in state chart, but every attempt to produce the trace from the state chart fails at a letter (input)
 - Otherwise return "confused"

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Example

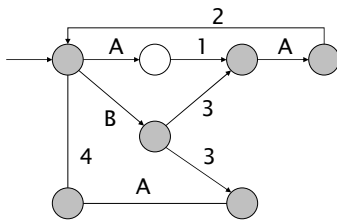
"A1C3"



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Example

"B3A4"



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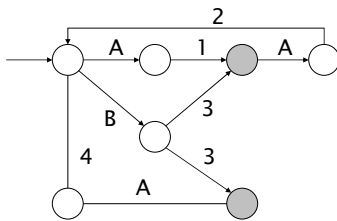
Step 2

- Create a state space explorer that computes for any given "pass" trace the set of those states where the specification state machine can be after the trace

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Example

"B3"



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Step 3

- Build a test execution loop:
 - Check observed trace
 - Compute current specification states
 - Choose an input that is valid in all the states
 - Send it to SUT
 - Receive response
 - Restart

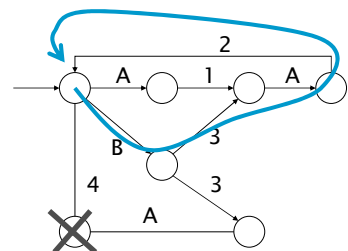
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Step 4

- Add testing heuristics
 - Co-operative planning
 - Adversarial planning
- Add test stopping heuristics
 - All states covered
 - "Seems" that no more states can be reached

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Example



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Step 5

- ▶ Augment the specification / system model with observed transition probabilities from the SUT
- ▶ Use these to guide test planning

- ▶ Investigate algorithms scalability

Next week

- ▶ Summary and conclusion
- ▶ Pre-examination (voluntary multiple-choice test, no effect on grade)