T-79.186 Reactive Systems: Temporal Logic LTL, part II

Spring 2005, Lecture 5

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Semantics in a Kripke Structure

The semantics in a Kripke structure can be (equivalently) defined as:

Definition 1 An LTL formula f holds in a Kripke structure M, denoted $M \models f$, iff for all paths π in M, such that $\pi_0 = s^0$, it holds that $\pi \models f$.

We also get the following:

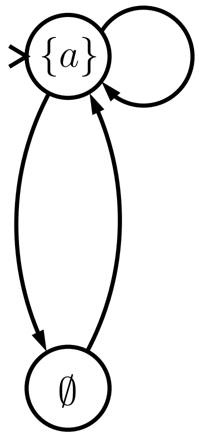
Definition 2 An LTL formula does not hold f in a Kripke structure M, denoted $M \not\models f$, iff there is a path π in M, such that $\pi_0 = s^0$ and $\pi \models \neg f$.

This definition is easier to handle, as to prove that f does not hold, we have to find one path where $\neg f$ holds. If such a path cannot be found, then we can conclude that f holds.

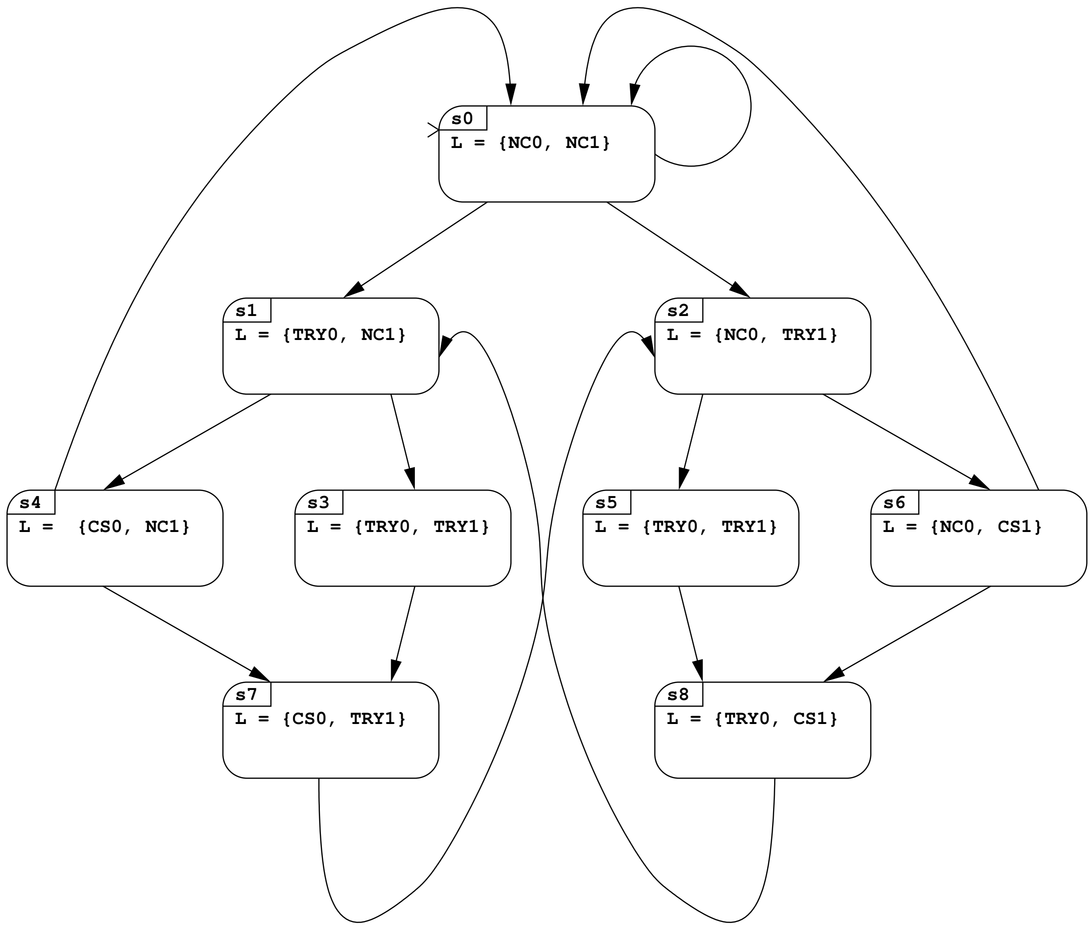
Universal Quantification

Note: It is possible that both $M \not\models f$ and $M \not\models \neg f$ hold.

Consider the formula $\Box a$ in the Kripke structure M below. We get that $M \not\models \Box a$. However, $\neg\Box a = \Diamond\neg a$, and thus it is also the case that $M \not\models \neg\Box a$.



Example: Kripke Structure M of a Mutex System



Example: Properties of the Mutex System

- $M \models \Box\neg(\mathit{CR0} \wedge \mathit{CR1})$
  (both processes are never in their critical sections at the same time)
- $M \models \Box(\mathit{TRY0} \Rightarrow \Diamond \mathit{CR0})$
  (always when process 0 enters the trying section, it is eventually followed by process 0 entering the critical section)
- $M \models \Box(\mathit{TRY0} \Rightarrow (\mathit{TRY0}\ U\ \mathit{CR0}))$
  (always when process 0 enters the trying section, it stays in the trying section until it enters the critical section)

Example: Unsatisfied Properties of the Mutex System

- $M \not\models \Diamond \mathit{CR0}$
  (the following does **not** hold: process 0 will eventually enter the critical section)
- $M \not\models \Box\Diamond \mathit{CR0} \Rightarrow \Box\Diamond \mathit{CR1}$
  (the following does **not** hold: if process 0 is infinitely often in the critical section, then also process 1 is infinitely often in the critical section)

Example: More Properties of the Mutex System

- $M \models \Box\Diamond \mathit{CR0} \Rightarrow \Box\Diamond \mathit{TRY0}$
  (if process 0 is infinitely often in the critical section, then it is also infinitely often in the trying section)
- $M \models \Diamond\Box \mathit{NC0} \Rightarrow \Diamond\Box\neg \mathit{CR0}$
  (if process 0 all the time from a certain point onward is in the non-critical section, then process 0 will all the time from a certain point onward be not in the critical section)
- $M \models (\Box\Diamond \mathit{TRY0} \wedge \Box\Diamond \mathit{TRY1}) \Rightarrow (\Box\Diamond \mathit{CR0} \wedge \Box\Diamond \mathit{CR1})$
  (if both process 0 and 1 are infinitely often in the trying section, then both process 0 and 1 are infinitely often also in the critical section)

LTL Property Patters: Scope

Quite often the requirements of a system follow some simple patterns. Sometimes we want to specify that a property should only hold in a certain context, called the **scope** of a property.

- **Global**: The scope of the property is the path.
- **Before** R: The scope of the property is all indexes which are strictly smaller than the first appearance of R.
- **After** Q: The scope of the property are all indexes greater or equal to the first appearance of Q.

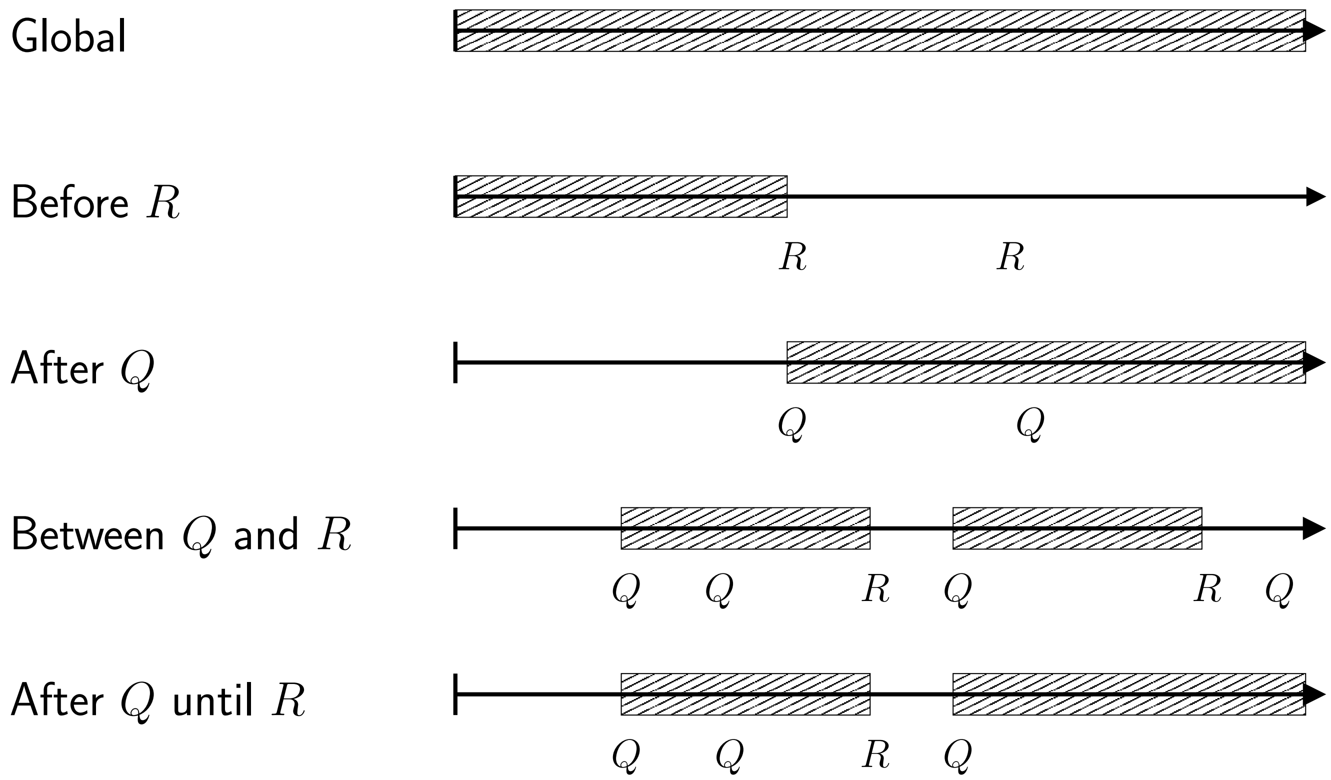
LTL Property Patters: Scope

- **Between Q and R**: The scope of the requirement contains all sequences of indexes, such that Q holds at the first index until (but not including) the larger index where R holds at the first time.

- **After Q until R**: The scope of the requirement contains all sequences of indexes, such that:
  (i) Q holds at the first index until (but not including) the larger index where R holds at the first time, or
  (ii) Q holds at the first index but the R never holds at a larger index.

Note: Scopes are define in a way that always includes the index at which the event triggering the scope happens, but excluding the index at which the event ending the scope happens.

LTL Property Patters: Scope



LTL Property Patters: Absence

Absence pattern specifies that "P is false" should hold within the scope:

- Global: $\Box(\neg P)$
- Before R: $(\Diamond R) \Rightarrow (\neg P \ U \ R)$
- After Q: $\Box(Q \Rightarrow (\Box(\neg P)))$
- Between Q and R: $\Box((Q \wedge \neg R \wedge \Diamond R) \Rightarrow (\neg P \ U \ R))$
- After Q until R: $\Box((Q \wedge \neg R) \Rightarrow (\neg P \ U \ (R \vee \Box(\neg P))))$

LTL Property Patters: Existence

Existence pattern specifies that "P becomes true" within the scope:

- Global: $\Diamond P$
- Before R: $\neg R\, U\, ((P \wedge \neg R) \vee (\Box \neg R))$
- After Q: $(\Box(\neg Q)) \vee (\Diamond(Q \wedge (\Diamond P)))$
- Between Q and R: $\Box((Q \wedge \neg R) \Rightarrow (\neg R\, U\, ((P \wedge \neg R) \vee (\Box(\neg R)))))$
- After Q until R: $\Box((Q \wedge \neg R) \Rightarrow (\neg R\, U\, (P \wedge \neg R)))$

LTL Property Patters

The LTL property patterns of the previous slides were included mainly to rehearse the semantics of LTL. However, they can be quite useful also in practice.

There are also other patterns available expressing:

- Universality: "P is true"-trivial to obtain from absence pattern
- Precedence: "S precedes P"
- Response: "S responds to P"
- Etc., etc.

These LTL property patterns can be obtained through the Web page:
`http://www.cis.ksu.edu/santos/spec-patterns/`

Relating Boolean and Temporal Operators

$$
\begin{aligned}
X(\varphi_1 \vee \varphi_2) &\equiv X\varphi_1 \ \vee \ X\varphi_2 \\
X(\varphi_1 \wedge \varphi_2) &\equiv X\varphi_1 \ \wedge \ X\varphi_2 \\
X\neg\varphi &\equiv \neg X\varphi
\end{aligned}
$$

$$
\begin{aligned}
\Diamond(\varphi_1 \vee \varphi_2) &\equiv \Diamond\varphi_1 \ \vee \ \Diamond\varphi_2 \\
\neg\Diamond\varphi &\equiv \Box\neg\varphi
\end{aligned}
$$

$$
\begin{aligned}
\Box(\varphi_1 \wedge \varphi_2) &\equiv \Box\varphi_1 \ \wedge \ \Box\varphi_2 \\
\neg\Box\varphi &\equiv \Diamond\neg\varphi
\end{aligned}
$$

$$
\begin{aligned}
(\varphi_1 \wedge \varphi_2)\, U\ \psi &\equiv (\varphi_1\ U\ \psi) \ \wedge \ (\varphi_2\ U\ \psi) \\
\varphi\ U\ (\psi_1 \vee \psi_2) &\equiv (\varphi\ U\ \psi_1) \ \vee \ (\varphi_1\ U\ \psi_2)
\end{aligned}
$$

Some Idempotence Laws

$$\begin{aligned}
\Diamond\Diamond\varphi &\equiv \Diamond\varphi \\
\Box\Box\varphi &\equiv \Box\varphi \\
\Diamond\Box\Diamond\varphi &\equiv \Box\Diamond\varphi \\
\Box\Diamond\Box\varphi &\equiv \Diamond\Box\varphi \\
\varphi\, U\, (\varphi\, U\, \psi) &\equiv \varphi\, U\, \psi
\end{aligned}$$

Unfolding Laws

$$
\begin{aligned}
\Diamond\varphi &\equiv \varphi \vee X\Diamond\varphi \\
\Box\varphi &\equiv \varphi \wedge X\Box\varphi \\
\varphi\, U\, \psi &\equiv \psi \vee (\varphi \wedge X(\varphi\, U\, \psi)) \\
\varphi\, R\, \psi &\equiv \psi \wedge (\varphi \vee X(\varphi\, R\, \psi))
\end{aligned}
$$