T-79.186 **Spring 2005**
Reactive Systems
Home Exercise 4
Deadline 4.4 16.15

Return your answers by email (Postscript or PDF) to Misa.Keinanen@hut.fi, or on paper to the lecture. Remember to include your name *and* student number.

For this home exercise round use the automata definition used in the lecture slides.

1.) For each LTL formula f_i below construct a Büchi automaton $\mathcal{A}_i$ that accepts the language $\{w \in (2^{AP_i})^\omega \mid w \models f_i\}$. In other words, the language contains exactly the infinite words which are models of the formula.

   (a) $AP_a = \{p\}$, $f_a = \Box\Diamond p$.
   (b) $AP_b = \{p\}$, $f_b = \Diamond\Box\neg p$.
   (c) $AP_c = \{p, q\}$, $f_c = p\mathbf{U}q$.
   (d) $AP_d = \{p, q\}$, $f_d = (\Diamond\Box p) \Rightarrow (\Diamond\Box q)$.
   (e) $AP_e = \{p\}$, $f_e = XXp$.
   (f) $AP_f = \{p, q\}$, $f_g = p\mathbf{R}q$.

2.) In the book (Bérard et al: Chapter 7.4, p. 87–89) the history variables method is described. The basic idea is to introduce a new Boolean variable h_i for each (past) temporal subformula, and initialize all them to **false** in the initial state. The model is instrumented to record changes in the truth of the past temporal subformulas following the semantics of past temporal operators.

   Let h_i' denote the value of the temporal subformula variable h_i in the previous time step, f_1, f_2 the values of variables corresponding to subformulas at the current time step, and finally f_1', f_2' the values of variables corresponding to subformulas at the previous time step.

   With this notation the update rule for the formula $h = \mathbf{X}^{-1} f_1$ becomes:
   $h_i := f_1'$. Give the update rules for all the other formula types:

   (a) $h_i = p$ for $p \in AP$,
   (b) $h_i = \neg f_1$,
   (c) $h_i = f_1 \vee f_2$,
   (d) $h_i = \mathbf{G}^{-1} f_1$, and
   (e) $f_1 \,\mathbf{S}\, f_2$.

3.) Consider the automaton of Figure 7.1 of the book (Bérard et al., p. 87). Add history variables to the model to model check a temporal formula containing past time temporal operators by using a standard CTL model checker. Also give the CTL formulas to model check in the following two cases.

(a) $\mathbf{AG}(\mathbf{X}^{-1}\mathit{alarm} \Rightarrow \mathbf{F}^{-1}\mathit{crash})$

(b) $\mathbf{AG}(\mathbf{F}^{-1}\mathit{alarm} \Rightarrow ((\mathit{crash} \vee \mathit{alarm})\,\mathbf{S}\,(\mathbf{X}^{-1}\mathit{ok})))$

Give the models with history variables added in the expressions in similar style to Figure 7.2, or notation similar to that of the exercise above.