1.) For each LTL formula $f_i$ below construct a Büchi automaton $A_i$ that accepts the language \( \{ w \in (2^{AP_i})^\omega \mid w \models f_i \} \). In other words, the language contains exactly the infinite words which are models of the formula.

(a) \( AP_a = \{ p \}, f_a = \Box \Diamond p \).
(b) \( AP_b = \{ p \}, f_b = \Diamond \Box \neg p \).
(c) \( AP_c = \{ p, q \}, f_c = p \mathbf{U} q \).
(d) \( AP_d = \{ p, q \}, f_d = (\Diamond \Box p) \Rightarrow (\Diamond q) \).
(e) \( AP_e = \{ p \}, f_e = \mathbf{X} \mathbf{X} p \).
(f) \( AP_f = \{ p, q \}, f_g = p \mathbf{R} q \).

2.) In the book (Bérard et al: Chapter 7.4, p. 87–89) the history variables method is described. The basic idea is to introduce a new Boolean variable $h_i$ for each (past) temporal subformula, and initialize all them to false in the initial state. The model is instrumented to record changes in the truth of the past temporal subformulas following the semantics of past temporal operators.

Let $h'_i$ denote the value of the temporal subformula variable $h_i$ in the previous time step, $f_1, f_2$ the values of variables corresponding to subformulas at the current time step, and finally $f'_1, f'_2$ the values of variables corresponding to subformulas at the previous time step.

With this notation the update rule for the formula $h = \mathbf{X}^{-1} f_1$ becomes:

\[ h_i := f'_1. \]

Give the update rules for all the other formula types:

(a) \( h_i = p \) for $p \in AP$,
(b) \( h_i = \neg f_1 \),
(c) \( h_i = f_1 \lor f_2 \),
(d) \( h_i = \mathbf{G}^{-1} f_1 \), and
(e) \( f_1 \mathbf{S} f_2 \).
3.) Consider the automaton of Figure 7.1 of the book (Bérand et al., p. 87). Add history variables to the model to model check a temporal formula containing past time temporal operators by using a standard CTL model checker. Also give the CTL formulas to model check in the following two cases.

(a) $\mathbf{AG}(X^{-1} \text{alarm} \Rightarrow F^{-1} \text{crash})$

(b) $\mathbf{AG}(F^{-1} \text{alarm} \Rightarrow ((\text{crash} \lor \text{alarm}) S (X^{-1} \text{ok})))$

Give the models with history variables added in the expressions in similar style to Figure 7.2, or notation similar to that of the exercise above.