1. Consider the three following finite state automata, where $\Sigma_1 = \Sigma = \{a, b\}$.

Automaton $A_1$:

![Automaton A1 diagram]

Automaton $A_2$:

![Automaton A2 diagram]

Automaton $A_3$:

![Automaton A3 diagram]

(a) Construct the finite state automaton $A_a = A_1 \cup A_2$.

(b) Construct the finite state automaton $A_b = A_1 \cap A_2$.

(c) Is the language accepted by $A_b$ non-empty? If not, give a word accepted by $A_b$. 
(d) Complement the deterministic automaton $A_1$, and give the resulting automaton $A_d$.

(e) Give a deterministic finite state automaton $A_e$, which accepts the same language as $A_3$.

(f) Describe the language accepted by $A_1$ as a function of the number of occurrences of $a$ and $b$. 