Project of T-79.179 in Spring 2005, PROD Part: Tic-Tac-Toe

The assumed rules of tic-tac-toe

The board consists of 9 positions, every position being denoted by a pair of integers between 1 and 3. A set of positions is a straight line of 3 positions if and only if the set is $\{\langle 1,1\rangle,\langle 1,2\rangle,\langle 1,3\rangle\}$ or $\{\langle 2,1\rangle,\langle 2,2\rangle,\langle 2,3\rangle\}$ or $\{\langle 3,1\rangle,\langle 3,2\rangle,\langle 3,3\rangle\}$ or $\{\langle 1,1\rangle,\langle 2,1\rangle,\langle 3,1\rangle\}$ or $\{\langle 1,2\rangle,\langle 2,2\rangle,\langle 3,2\rangle\}$ or $\{\langle 1,3\rangle,\langle 2,3\rangle,\langle 3,3\rangle\}$ or $\{\langle 1,1\rangle,\langle 2,2\rangle,\langle 3,3\rangle\}$ or $\{\langle 1,2\rangle,\langle 2,2\rangle,\langle 3,2\rangle\}$. There are two players: the x player and the o player. At any time, every position of the board is in one and only one of the states _ (blank), x and o. In the initial configuration, every position is in the state _.

The turns in a game are numbered by consecutive integers, the first turn being numbered by 1. If the game is not over and the number of a turn is not divisible by 2, then the action of the turn is that the x player switches the state of one and only one position from $_{-}$ to x. If the game is not over and the number of a turn is divisible by 2, then the action of the turn is that the o player switches the state of one and only one position from $_{-}$ to o.

The x player wins a game immediately whenever the game reaches a configuration where some straight line of 3 positions has all of its positions in the state x. The o player wins a game immediately whenever the game reaches a configuration where some straight line of 3 positions has all of its positions in the state o. There is no other way to win a game.

A draw is obtained for a game immediately whenever the game reaches a configuration where every straight line of 3 positions has at least one position in the state x and at least one position in the state o. There is no other way to obtain a draw for a game.

A game is over immediately whenever one of the players wins the game or whenever a draw is obtained for the game. There is no other way for the game to be over.

The assignment

Make a high-level Petri net that models all possible sequences of actions in tic-tac-toe. The net should be expressed as a net description file written in the net description language of PROD. Let us assume the definitions given on page 29 of PROD Reference Manual, still taking into account the redefinitions on page 9 of the up-to-date addendum. The reachability graph of the net should be such that the number of nodes is equal to the number of strongly connected components, every win configuration is represented by one and only one terminal node, every terminal node represents one and only one win configuration, every draw configuration is represented by one and only one nontrivial terminal strongly connected component, and every nontrivial terminal strongly connected component represents one and only one draw configuration.

Send the net description file and a verbal report to the Firstname.Lastname@hut.fi e-mail address of Kimmo Varpaaniemi, not later than May 9, 2005. Appropriate additional files may have to be included in that e-mail message if the net description file contains an #include directive or uses a function or a global variable that is not defined in the net description file.

Further information

PROD Reference Manual, its errata and the up-to-date addendum are available via http://www.tcs.hut.fi/Studies/T-79.179/. You do not have to download or install PROD. If you do not have a user account in the Unix machines of HUT Computing Centre, you should easily get such an account by utilising http://www.hut.fi/cc/support/. In any Unix machine of HUT Computing Centre, if mynet.net is the name of the net description file, then the command sequence

use -q prod

rhjttt.sh mynet.net

should produce the file mynet.txt that not only mentions the number of nodes and the number of strongly connected components but also very explicitly lists the terminal nodes and the nontrivial terminal strongly connected components. Repetition of the above use command is not necessary, so you might consider adding that command to the end of your default shell initialisation file. So, it is technically possible to proceed as if rhjttt.sh were the only interface to PROD. Since rhjttt.sh is a shell script, the command

cat `which rhjttt.sh`

may be helpful.