7.1.1 Assume renaming function \( f : A \rightarrow A \) with \( f(a) = c \) and \( f(b) = c \). Derive
\[
\rho_f(\langle X | X = aX + bX \rangle) = \langle Y | Y = cY \rangle
\]
from the axioms.

7.2.2 Let \( A = \{push, on, off\} \) and \( S = \{0, 1\} \), where intuitively state 0 represents that some machine is off, and state 1 that this same machine is on. Use the state operator to specify a button, such that pushing this button alternately turns the machine on and off. That is, define mappings \( action : S \times A \rightarrow A \) and \( effect : S \times A \rightarrow S \) such that
\[
\lambda_0(\langle X | X = push \cdot X \rangle) = on \cdot off \cdot \lambda_0(\langle X | X = push \cdot X \rangle).
\]
Derive the equation above from the axioms for the state operator, using your definitions for the mappings \( action \) and \( effect \).

7.3.3 Let \( b < c \), \( a < \tau \), \( b < \tau \), and \( c < \tau \). Derive the equation
\[
\Theta(a(\tau(b + c) + b)) = \Theta(a(b + c))
\]
from the axioms.