T-79.179 **Spring 2005**
Parallel and Distributed Digital Systems
Exercise 1 Solutions
24-30.1.2005

The exercise numbering is identical to the book [Fok00].

2.1.1 $a{\cdot}(b+c)$ and $(a\cdot b)+(a\cdot c)$

2.2.1. Process graph:

$$\begin{array}{c} \{((a+b)\cdot(a+c))\cdot d \xrightarrow{a} (a+c)\cdot d, \\ ((a+b)\cdot(a+c))\cdot d \xrightarrow{b} (a+c)\cdot d, \\ (a+c)\cdot d \xrightarrow{a} d, \\ (a+c)\cdot d \xrightarrow{c} d, \\ d \xrightarrow{d} \surd\} \end{array}$$

Derivations:

$$\begin{array}{c|l} a \xrightarrow{a} \surd & (\frac{}{v\xrightarrow{v}\surd}, v{:=}a) \\ \hline (a+b) \xrightarrow{a} \surd & (\frac{x\xrightarrow{v}\surd}{x+y\xrightarrow{v}\surd}, v{:=}a, x{:=}a, y{:=}b) \\ \hline ((a+b)\cdot(a+c)) \xrightarrow{a} (a+c) & (\frac{x\xrightarrow{v}\surd}{x\cdot y\xrightarrow{v}y}, v{:=}a, x{:=}(a+b), y{:=}(a+c)) \\ \hline ((a+b)\cdot(a+c))\cdot d \xrightarrow{a} (a+c)\cdot d & (\frac{x\xrightarrow{v}x'}{x\cdot y\xrightarrow{v}x'\cdot y}, v{:=}a, x{:=}((a+b)\cdot(a+c)), \\ & y{:=}d, x'{:=}(a+c)) \end{array}$$

$$\begin{array}{c|l} b \xrightarrow{b} \surd & (\frac{}{v\xrightarrow{v}\surd}, v{:=}b) \\ \hline (a+b) \xrightarrow{b} \surd & (\frac{y\xrightarrow{v}\surd}{x+y\xrightarrow{v}y}, v{:=}b, x{:=}a, y{:=}b) \\ \hline ((a+b)\cdot(a+c)) \xrightarrow{b} (a+c) & (\frac{x\xrightarrow{v}\surd}{x\cdot y\xrightarrow{v}y}, v{:=}b, x{:=}(a+b), y{:=}(a+c)) \\ \hline ((a+b)\cdot(a+c))\cdot d \xrightarrow{b} (a+c)\cdot d & (\frac{x\xrightarrow{v}x'}{x\cdot y\xrightarrow{v}x'\cdot y}, v{:=}b, x{:=}((a+b)\cdot(a+c)), \\ & y{:=}d, x'{:=}(a+c)) \end{array}$$

$$\begin{array}{c|l} a \xrightarrow{a} \surd & (\frac{}{v\xrightarrow{v}\surd}, v{:=}a) \\ \hline (a+c) \xrightarrow{a} \surd & (\frac{x\xrightarrow{v}\surd}{x+y\xrightarrow{v}\surd}, v{:=}a, x{:=}a, y{:=}c) \\ \hline (a+c)\cdot d \xrightarrow{a} d & (\frac{x\xrightarrow{v}\surd}{x\cdot y\xrightarrow{v}y}, v{:=}a, x{:=}(a+c), y{:=}d \end{array}$$

$$\begin{array}{c|l} c \xrightarrow{c} \surd & (\frac{}{v\xrightarrow{v}\surd}, v{:=}c) \\ \hline (a+c) \xrightarrow{c} \surd & (\frac{y\xrightarrow{v}\surd}{x+y\xrightarrow{v}\surd}, v{:=}c, x{:=}a, y{:=}c) \\ \hline (a+c)\cdot d \xrightarrow{c} d & (\frac{x\xrightarrow{v}\surd}{x\cdot y\xrightarrow{v}y}, v{:=}c, x{:=}(a+c), y{:=}d \end{array}$$

$$d \xrightarrow{d} \surd \quad (\frac{}{v\xrightarrow{v}\surd}, v{:=}d)$$

2.3.1

- Yes. The bisimulation relation that relates the two basic process terms $(b+c)a+ba+ca$ and $ba+ca$ is:

$$\{(b+c)a+ba+ca\mathcal{B}ba+ca, a\mathcal{B}a\}.$$

- No, the process terms $a(b+c)+ab+ac$ and $ab+ac$ are not bisimilar.

Consider the process graphs

$$\{a(b+c)+ab+ac \xrightarrow{a} b+c,$$

$$a(b+c)+ab+ac \xrightarrow{a} b,$$

$$a(b+c)+ab+ac \xrightarrow{a} c,$$

$$b+c \xrightarrow{b} \surd,$$

$$b+c \xrightarrow{c} \surd,$$

$$b \xrightarrow{b} \surd,$$

$$c \xrightarrow{c} \surd\}$$

and

$$\{ab+ac \xrightarrow{a} b,$$

$$ab+ac \xrightarrow{a} c,$$

$$b \xrightarrow{b} \surd,$$

$$c \xrightarrow{c} \surd\}.$$

It is seen that the node $b+c$ in the former graph cannot be related with any node of the latter such that all conditions 1-4 in Def. 2.3.1 hold.

- Yes. The bisimulation relation that relates the two basic process terms $(a+a)(bc)+(ab)(c+c)$ and $(a(b+b))(c+c)$ is:

$$\{(a+a)(bc)+(ab)(c+c)\mathcal{B}(a(b+b))(c+c),$$

$$bc\mathcal{B}(b+b)(c+c),$$

$$b(c+c)\mathcal{B}(b+b)(c+c),$$

$$c\mathcal{B}(c+c),$$

$$(c+c)\mathcal{B}(c+c)\}.$$

2.3.3 Show by induction that a^k and a^{k+1} cannot be related by a bisimulation relation.

Base case:
$a \xrightarrow{a} \surd$, while aa cannot terminate successfully by the execution of an a-transition. Hence, a and aa are not bisimilar.

Inductive case:
$a^{k+1} \xrightarrow{a} a^k$ is the only transition of a^{k+1}, while $a^{k+2} \xrightarrow{a} a^{k+1}$ is the only transition of a^{k+2}.

Hence, a^{k+1} and a^{k+2} are not bisimilar.

2.4.1 The proof consists of three parts and proceeds in the following way.

$(A2' \wedge A3 \Rightarrow A1)$:

$$x+y \overset{A3}{=} (x+y)+(x+y) \overset{A2'}{=} y+((x+y)+x) \overset{A2'}{=} y+(y+(x+x)) \overset{A2'}{=}$$

$$((x+x)+y)+y \overset{A2'}{=} (x+(y+x))+y \overset{A2'}{=} (y+x)+(y+x) \overset{A3}{=} y+x$$

$(A2' \wedge A1 \Rightarrow A2)$:

$$(x+y)+z \overset{A2'}{=} y+(z+x) \overset{A1}{=} (z+x)+y \overset{A2'}{=} x+(y+z)$$

$(A1 \wedge A2 \Rightarrow A2')$:

$$(x+y)+z \overset{A1}{=} z+(x+y) \overset{A1}{=} z+(y+x) \overset{A2}{=}$$

$$(z+y)+x \overset{A1}{=} (y+z)+x \overset{A2}{=} y+(z+x)$$