T-79.179 **Spring 2005**
Parallel and Distributed Digital Systems
Exercise 3 Solutions
21-27.2.2005

4.1.2

- Both equations $X = aX$ and $Y = bX$ are guarded.
- Equation $Y = aX$ is already guarded.
  Equation $X = Y$ can be transformed to guarded form by replacing the right-hand side Y by the right-hand side aX of $Y = aX$; thus, we obtain $X = aX$ which is guarded.
- We will use axioms

$$LM2 : v \between y = v \cdot y$$

$$LM4 : (x + y) \between z = x \between z + y \between z$$

for the left merge operator $\between$.
The right-hand side term $(a+b) \between X$ of equation $X = (a+b) \between X$ can be transformed to guarded form by the following applications of $LM2$ and $LM4$:

$$(a + b) \between X \stackrel{LM4}{=}$$

$$a \between X + b \between X \stackrel{LM2}{=}$$

$$a \cdot X + b \between X \stackrel{LM2}{=}$$

$$a \cdot X + b \cdot X$$

Thus, we obtain $X = a \cdot X + b \cdot X$ which is guarded.

4.2.1 The derivation is:

$$\frac{b \stackrel{b}{\to} \surd}{b\langle X|E\rangle \stackrel{b}{\to} \langle X|E\rangle} \; (\tfrac{}{v \stackrel{v}{\to} \surd}, v{:=}b)$$

$$\frac{b\langle X|E\rangle \stackrel{b}{\to} \langle X|E\rangle}{\langle Y|E\rangle \stackrel{b}{\to} \langle X|E\rangle} \; (\tfrac{x \stackrel{v}{\to} \surd}{xy \stackrel{v}{\to} y}, v{:=}b, x{:=}b, y{:=}\langle X|E\rangle)$$

$$(\tfrac{b\langle X|E\rangle \stackrel{v}{\to} y}{\langle Y|E\rangle \stackrel{v}{\to} y}, v{:=}b, y{:=}\langle X|E\rangle)$$

4.4.1 Let E be the recursive specification:

$$\{X = aX + bY, Y = cX + aY\}.$$

Specification E is a linear recursive specification because both of its recursive equations are of the form

$$X = a_1X_1 + \cdots + a_kX_k + b_1 + \cdots + b_l$$

with recursion variables $X, X_1, \ldots, X_k$ and atomic actions $a_1, \ldots, a_k, b_1, \ldots, b_l$.
Consider process term $\langle X|E\rangle$. Its process graph is

$$\{\langle X|E\rangle \stackrel{a}{\to} \langle X|E\rangle, \langle X|E\rangle \stackrel{b}{\to} \langle Y|E\rangle, \langle Y|E\rangle \stackrel{c}{\to} \langle X|E\rangle, \langle Y|E\rangle \stackrel{a}{\to} \langle Y|E\rangle\}$$

which is bisimilar to the given graph. Here, the bisimulation relation is:

$$\{s_0 \mathcal{B} \langle X|E\rangle, s_1 \mathcal{B} \langle Y|E\rangle\}.$$