T-79.179 **Spring 2005**
Parallel and Distributed Digital Systems
Exercise 3 Solutions
21-27.2.2005

4.1.2

- Both equations $X = aX$ and $Y = bX$ are guarded.
- Equation $Y = aX$ is already guarded.
  Equation $X = Y$ can be transformed to guarded form by replacing the right-hand side Y by the right-hand side aX of $Y = aX$; thus, we obtain $X = aX$ which is guarded.
- We will use axioms

$$LM2 : v \mathbin{\underline{\|}} y = v \cdot y$$

$$LM4 : (x + y) \mathbin{\underline{\|}} z = x \mathbin{\underline{\|}} z + y \mathbin{\underline{\|}} z$$

  for the left merge operator $\mathbin{\underline{\|}}$.
  The right-hand side term $(a+b) \mathbin{\underline{\|}} X$ of equation $X = (a+b) \mathbin{\underline{\|}} X$ can be transformed to guarded form by the following applications of $LM2$ and $LM4$:

$$(a+b) \mathbin{\underline{\|}} X \overset{LM4}{=}$$

$$a \mathbin{\underline{\|}} X + b \mathbin{\underline{\|}} X \overset{LM2}{=}$$

$$a \cdot X + b \mathbin{\underline{\|}} X \overset{LM2}{=}$$

$$a \cdot X + b \cdot X$$

  Thus, we obtain $X = a \cdot X + b \cdot X$ which is guarded.

4.2.1 The derivation is:

$$\dfrac{\dfrac{b \xrightarrow{b} \surd}{b\langle X|E\rangle \xrightarrow{b} \langle X|E\rangle}}{\langle Y|E\rangle \xrightarrow{b} \langle X|E\rangle}$$

- $b \xrightarrow{b} \surd$ $\quad (\frac{}{v \xrightarrow{v} \surd}, v{:=}b)$
- $b\langle X|E\rangle \xrightarrow{b} \langle X|E\rangle$ $\quad (\frac{x \xrightarrow{v} \surd}{xy \xrightarrow{v} y}, v{:=}b, x{:=}b, y{:=}\langle X|E\rangle)$
- $\langle Y|E\rangle \xrightarrow{b} \langle X|E\rangle$ $\quad (\frac{b\langle X|E\rangle \xrightarrow{v} y}{\langle Y|E\rangle \xrightarrow{v} y}, v{:=}b, y{:=}\langle X|E\rangle)$

4.4.1 Let E be the recursive specification:

$$\{X = aX + bY, Y = cX + aY\}.$$

Specification E is a linear recursive specification because both of its recursive equations are of the form

$$X = a_1X_1 + \cdots + a_kX_k + b_1 + \cdots + b_l$$

with recursion variables $X, X_1, \ldots, X_k$ and atomic actions $a_1, \ldots, a_k, b_1, \ldots, b_l$. Consider process term $\langle X|E\rangle$. Its process graph is

$$\{\langle X|E\rangle \xrightarrow{a} \langle X|E\rangle, \langle X|E\rangle \xrightarrow{b} \langle Y|E\rangle, \langle Y|E\rangle \xrightarrow{c} \langle X|E\rangle, \langle Y|E\rangle \xrightarrow{a} \langle Y|E\rangle\}$$

which is bisimilar to the given graph. Here, the bisimulation relation is:

$$\{s_0\mathcal{B}\langle X|E\rangle, s_1\mathcal{B}\langle Y|E\rangle\}.$$