**T-79.179** **Spring 2005**
**Parallel and Distributed Digital Systems**
**Exercise 6 Solutions**
**11-17.4.2005**

**7.1.1** We will use $RDP$ and $RSP$ (see p. 36 in [Fok00]) together with the following axioms:

$$RN1 : \rho_f(v) = f(v)$$

$$RN3 : \rho_f(x + y) = \rho_f(x) + \rho_f(y)$$

$$RN4 : \rho_f(x \cdot y) = \rho_f(x) \cdot \rho_f(y)$$

$$A3 : x + x = x$$

The derivation is:

$$\rho_f(\langle X|X = aX + bX\rangle) \stackrel{RDP}{=}$$

$$\rho_f(a\langle X|X = aX + bX\rangle + b\langle X|X = aX + bX\rangle) \stackrel{RN3}{=}$$

$$\rho_f(a\langle X|X = aX + bX\rangle) + \rho_f(b\langle X|X = aX + bX\rangle) \stackrel{RN4}{=}$$

$$\rho_f(a)\rho_f(\langle X|X = aX + bX\rangle) + \rho_f(b\langle X|X = aX + bX\rangle) \stackrel{RN4}{=}$$

$$\rho_f(a)\rho_f(\langle X|X = aX + bX\rangle) + \rho_f(b)\rho_f(\langle X|X = aX + bX\rangle) \stackrel{RN1}{=}$$

$$c\rho_f(\langle X|X = aX + bX\rangle) + \rho_f(b)\rho_f(\langle X|X = aX + bX\rangle) \stackrel{RN1}{=}$$

$$c\rho_f(\langle X|X = aX + bX\rangle) + c\rho_f(\langle X|X = aX + bX\rangle) \stackrel{A3}{=}$$

$$c\rho_f(\langle X|X = aX + bX\rangle)$$

Thus, by RSP, $\rho_f(\langle X|X = aX + bX\rangle) = \langle Y = Y|cY\rangle$.

**7.2.2** We will use $RDP$ (see p. 36 in [Fok00]) and the following axiom

$$SO4 : \lambda_s(v \cdot y) = action(s, v) \cdot \lambda_{effect(s,v)}(y)$$

for the state operator.



Define *action* and *push* as follows:

$$action(0, push) = on$$

$$action(1, push) = off$$

$$effect(0, push) = 1$$

$$effect(1, push) = 0$$

Then, the derivation is:

$$\lambda_0(\langle X|X = push \cdot X\rangle) \stackrel{RDP}{=}$$

$$\lambda_0(push \cdot \langle X|X = push \cdot X\rangle) \stackrel{SO4}{=}$$

$$on \cdot \lambda_1(\langle X|X = push \cdot X\rangle) \stackrel{RDP}{=}$$

$$on \cdot \lambda_1(push \cdot \langle X|X = push \cdot X\rangle) \stackrel{SO4}{=}$$

$$on \cdot off \cdot \lambda_0(\langle X|X = push \cdot X\rangle)$$

**7.3.3** We will use axioms:

$$TH1 : \Theta(v) = v$$

$$TH3 : \Theta(x + y) = \Theta(x) \triangleleft y + \Theta(y) \triangleleft x$$

$$TH4 : \Theta(x \cdot y) = \Theta(x) \cdot \Theta(y)$$

$$P1 : v \not< w \qquad v \triangleleft w = v$$

$$P2 : v < w \qquad v \triangleleft w = \delta$$

$$P6 : (x \cdot y) \triangleleft z = (x \triangleleft z) \cdot y$$

$$P8 : x \triangleleft (y \cdot z) = x \triangleleft z$$

$$A1 : x + y = y + x$$

$$A5 : x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$$A6 : x + \delta = x$$

$$B1 : v \cdot \tau = v$$

We first derive $\Theta(a(b+c)) = ac$:

$$\Theta(a(b+c)) \overset{TH4}{=}$$
$$\Theta(a)\Theta(b+c) \overset{TH1}{=}$$
$$a\Theta(b+c) \overset{TH3}{=}$$
$$a(\Theta(b) \triangleleft c + \Theta(c) \triangleleft b) \overset{TH1}{=}$$
$$a(b \triangleleft c + c \triangleleft b) \overset{P1}{=}$$
$$a(b \triangleleft c + c) \overset{P2}{=}$$
$$a(\delta + c) \overset{A1}{=}$$
$$a(c + \delta) \overset{A6}{=}$$
$$ac$$

Then, we derive $\Theta(a(\tau(b+c)+b)) = ac$:

$$\Theta(a(\tau(b+c)+b)) \overset{TH4}{=}$$
$$\Theta(a)\Theta(\tau(b+c)+b) \overset{TH1}{=}$$
$$a\Theta(\tau(b+c)+b) \overset{TH3}{=}$$
$$a(\Theta(\tau(b+c)) \triangleleft b + \Theta(b) \triangleleft (\tau(b+c))) \overset{TH1}{=}$$
$$a(\Theta(\tau(b+c)) \triangleleft b + b \triangleleft (\tau(b+c))) \overset{TH4}{=}$$
$$a((\Theta(\tau)\Theta(b+c)) \triangleleft b + b \triangleleft (\tau(b+c))) \overset{TH1}{=}$$
$$a((\tau\Theta(b+c)) \triangleleft b + b \triangleleft (\tau(b+c))) \overset{P8}{=}$$
$$a((\tau\Theta(b+c)) \triangleleft b + b \triangleleft \tau) \overset{TH3}{=}$$
$$a((\tau(\Theta(b) \triangleleft c + \Theta(c) \triangleleft b)) \triangleleft b + b \triangleleft \tau) \overset{TH1}{=}$$
$$a((\tau(b \triangleleft c + \Theta(c) \triangleleft b)) \triangleleft b + b \triangleleft \tau) \overset{TH1}{=}$$
$$a((\tau(b \triangleleft c + c \triangleleft b)) \triangleleft b + b \triangleleft \tau) \overset{P2}{=}$$
$$a((\tau(b \triangleleft c + c \triangleleft b)) \triangleleft b + \delta \overset{A6}{=}$$
$$a((\tau(b \triangleleft c + c \triangleleft b)) \triangleleft b \overset{P2}{=}$$
$$a((\tau(\delta + c \triangleleft b)) \triangleleft b \overset{P1}{=}$$
$$a((\tau(\delta + c)) \triangleleft b \overset{A1}{=}$$
$$a((\tau(c + \delta)) \triangleleft b \overset{A6}{=}$$
$$a((\tau(c)) \triangleleft b \overset{P6}{=}$$
$$a((\tau \triangleleft b)c) \overset{P1}{=}$$
$$a(\tau c) \overset{A5}{=}$$
$$(a\tau)c \overset{B1}{=}$$
$$ac$$