T-79.179 **Spring 2005**
Parallel and Distributed Digital Systems
Exercise 4 Solutions
7-13.3 2005

5.1.1 The branching bisimulation that relates the terms a and $a\tau$ is:

$$\{a\mathcal{B}a\tau, \surd\mathcal{B}\tau, \surd\mathcal{B}\surd\}$$

The branching bisimulation that relates the terms a and τa is:

$$\{a\mathcal{B}\tau a, a\mathcal{B}a, \surd\mathcal{B}\surd\}$$

The branching bisimulation that relates the terms $a\tau$ and τa is:

$$\{a\tau\mathcal{B}\tau a, a\tau\mathcal{B}a, \tau\mathcal{B}\surd, \surd\mathcal{B}\surd\}$$

5.1.2 The branching bisimulation that relates the terms is:

$\{\ \tau(\tau(a+b)+b)+a\mathcal{B}a+b,$
$\tau(a+b)+b\mathcal{B}a+b,$
$a+b\mathcal{B}a+b,$
$\surd\mathcal{B}\surd\ \}$

5.4.1
$$\frac{\partial_{\{a,b\}}(aa\|bb) \xrightarrow{c} \partial_{\{a,b\}}(a\|b))}{\tau_{\{c\}}(\partial_{\{a,b\}}(aa)\|(bb)) \xrightarrow{\tau} \tau_{\{c\}}(\partial_{\{a,b\}}(a\|b))} \quad (\tfrac{x \xrightarrow{v} x'}{\tau_I(x) \xrightarrow{\tau} \tau_I(x')}, v \in I, x{:=}\partial_{\{a,b\}}(aa\|bb), x'{:=}\partial_{\{a,b\}}(a\|b), v{:=}c, c \in \{c\})$$

$$\frac{((aa)\|(bb)) \xrightarrow{c} (a\|b)}{\partial_{\{a,b\}}(((aa)\|(bb)) \xrightarrow{c} (a\|b))} \quad (\tfrac{x \xrightarrow{v} x'}{\partial_H(x) \xrightarrow{v} \partial_H(x')}, v \notin H, x{:=}((aa)\|(bb)), x'{:=}(a\|b), v{:=}c)$$

$$\frac{aa \xrightarrow{a} a \quad bb \xrightarrow{b} b}{((aa)\|(bb)) \xrightarrow{c} (a\|b)} \quad (\tfrac{x \xrightarrow{v} x' \quad y \xrightarrow{w} y'}{x\|y \xrightarrow{\gamma(v,w)} x'\|y'}, x{:=}aa, x'{:=}a, y{:=}bb, y'{:=}b, v{:=}a, w{:=}b, \gamma(a,b) = c)$$

$$\frac{a \xrightarrow{a} \surd}{aa \xrightarrow{a} a} \quad (\tfrac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, x{:=}a, y{:=}a, v{:=}a)$$

$$\frac{b \xrightarrow{b} \surd}{bb \xrightarrow{b} b} \quad (\tfrac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, x{:=}b, y{:=}b, v{:=}b)$$

$$b \xrightarrow{b} \surd \quad (\tfrac{}{v \xrightarrow{v} \surd}, v{:=}b)$$

$$a \xrightarrow{a} \surd \quad (\tfrac{}{v \xrightarrow{v} \surd}, v{:=}a)$$

5.4.2 The branching bisimulation which relates the terms $\tau_{\{a\}}(\langle X \mid X = aX\rangle)$ and δ is:

$$\{\tau_{\{a\}}(\langle X \mid X = aX\rangle)\ \mathcal{B}\ \delta\}.$$

To see this, notice that the process graph of $\tau_{\{a\}}(\langle X \mid X = aX\rangle)$ is

$$\{\langle X \mid X = aX\rangle \xrightarrow{\tau} \langle X \mid X = aX\rangle\}$$

and the process graph of δ is

$$\{\}.$$

It is straightforward to verify conditions 1-2 of Definition 5.1.1. To verify conditions 3-4 of Definition 5.1.1, notice that termination predicate $\downarrow$ holds for neither $\langle X \mid X = aX\rangle$ nor δ. Therefore, the implication becames trivially true in both conditions 3 and 4.