T-79.179 **Spring 2005**
Parallel and Distributed Digital Systems
Exercise 2 Solutions
7-13.1.2005

3.1.1. Given the communication function $\gamma(a,a) = \gamma(a,b) = \gamma(b,b) = c$ the process graph is:

$$
\begin{gathered}
\{(((ab)a) \parallel b) \xrightarrow{a} ((ba) \parallel b),\\
(((ab)a) \parallel b) \xrightarrow{c} (ba),\\
(((ab)a) \parallel b) \xrightarrow{b} (aba),\\
((ba \parallel b) \xrightarrow{b} (a \parallel b),\\
((ba \parallel b) \xrightarrow{c} (a),\\
((ba \parallel b) \xrightarrow{b} (ba),\\
(a \parallel b) \xrightarrow{a} (b),\\
(a \parallel b) \xrightarrow{c} \surd,\\
(a \parallel b) \xrightarrow{b} (a),\\
((ab)a) \xrightarrow{a} (ba),\\
(ba \xrightarrow{b} (a),\\
b \xrightarrow{b} \surd,\\
a \xrightarrow{a} \surd\}
\end{gathered}
$$

The derivations are:

$$\frac{aba \xrightarrow{a} ba}{aba \parallel b \xrightarrow{a} ba \| b} \quad \left(\frac{x \xrightarrow{v} x'}{x \parallel y \xrightarrow{v} x' \parallel y}, x{:=}aba, y{:=}b, x'{:=}ba, v{:=}a\right)$$

$$\frac{aba \xrightarrow{a} ba \quad b \xrightarrow{b} \surd}{aba \parallel b \xrightarrow{c} ba} \quad \left(\frac{x \xrightarrow{v} x' y \xrightarrow{w} \surd}{x \parallel y \xrightarrow{\gamma(v,w)} x'}, x{:=}aba, y{:=}b, x'{:=}ba, v{:=}a, w{:=}b\right)$$

$$\frac{b \xrightarrow{b} \surd}{aba \parallel b \xrightarrow{b} aba} \quad \left(\frac{y \xrightarrow{v} \surd}{x \parallel y \xrightarrow{v} x}, x{:=}aba, y{:=}b, v{:=}b\right)$$

$$\frac{b \xrightarrow{b} \surd}{ba \parallel b \xrightarrow{b} ba} \quad \left(\frac{y \xrightarrow{v} \surd}{x \parallel y \xrightarrow{v} x}, x{:=}ba, y{:=}b, v{:=}b\right)$$

$$\frac{ba \xrightarrow{b} a \quad b \xrightarrow{b} \surd}{ba \parallel b \xrightarrow{c} a} \quad \left(\frac{x \xrightarrow{v} x' y \xrightarrow{w} \surd}{x \parallel y \xrightarrow{\gamma(v,w)} x'}, x{:=}ba, x'{:=}a, y{:=}b, v{:=}b, w{:=}b\right)$$

$$\frac{ba \xrightarrow{b} a}{ba \parallel b \xrightarrow{b} a \parallel b} \quad \left(\frac{x \xrightarrow{v} x'}{x \parallel y \xrightarrow{v} x' \| y}, x{:=}ba, y{:=}b, v{:=}b, x'{:=}a\right)$$

$$\frac{b \xrightarrow{b} \surd}{a \parallel b \xrightarrow{b} a} \quad \left(\frac{y \xrightarrow{v} \surd}{x \parallel y \xrightarrow{v} x}, x{:=}a, y{:=}b, v{:=}b\right)$$

$$\frac{a \xrightarrow{a} \surd \quad b \xrightarrow{b} \surd}{a \parallel b \xrightarrow{c} \surd} \quad \left(\frac{x \xrightarrow{v} \surd \; y \xrightarrow{w} \surd}{x \parallel y \xrightarrow{\gamma(v,w)} \surd}, x{:=}a, y{:=}b, v{:=}a, w{:=}b\right)$$

$$\frac{a \xrightarrow{a} \surd}{a \parallel b \xrightarrow{a} b} \quad \left(\frac{x \xrightarrow{v} \surd}{x \parallel y \xrightarrow{v} y}, v{:=}a, x{:=}a, y{:=}b\right)$$

$$\frac{a \xrightarrow{a} \surd}{aba \xrightarrow{a} ba} \quad \left(\frac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, v{:=}a, x{:=}a, y{:=}ba\right)$$

$$\frac{b \xrightarrow{b} \surd}{ba \xrightarrow{b} a} \quad \left(\frac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, v{:=}b, x{:=}b, y{:=}a\right)$$

$$b \xrightarrow{b} \surd \quad \left(\frac{}{v \xrightarrow{v} \surd}, v{:=}b\right)$$

$$a \xrightarrow{a} \surd \quad \left(\frac{}{v \xrightarrow{v} \surd}, v{:=}a\right)$$

3.4.2 The process graphs and derivations are:

- $\{\partial_{\{a\}}(ac)\}$

- $\{\partial_{\{a\}}((a+b)c) \xrightarrow{b} \partial_{\{a\}}(c), \partial_{\{a\}}(c) \xrightarrow{c} \surd\}$

$$\dfrac{\dfrac{\dfrac{\overline{b \xrightarrow{b} \surd}}{(a+b) \xrightarrow{b} \surd}}{(a+b)c \xrightarrow{b} c}}{\partial_{\{a\}}((a+b)c) \xrightarrow{b} \partial_{\{a\}}(c)}
\qquad
\begin{array}{l}
\left(\frac{}{v \xrightarrow{v} \surd}, v{:=}b\right) \\
\left(\frac{y \xrightarrow{v} \surd}{x+y \xrightarrow{v} \surd}, x{:=}a, y{:=}b, v{:=}b\right) \\
\left(\frac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, x{:=}(a+b), y{:=}c, v{:=}b\right) \\
\left(\frac{x \xrightarrow{v} x'}{\partial_H(x) \xrightarrow{v} \partial_H(x')} v \notin H, x{:=}(a+b)c, x'{:=}c, v{:=}b, H{:=}\{a\}\right)
\end{array}$$

$$\dfrac{\overline{c \xrightarrow{c} \surd}}{\partial_{\{a\}}(c) \xrightarrow{c} \surd}
\qquad
\begin{array}{l}
\left(\frac{}{v \xrightarrow{v} \surd}, v{:=}c\right) \\
\left(\frac{x \xrightarrow{v} \surd}{\partial_H(x) \xrightarrow{v} \surd} v \notin H, x{:=}c, v{:=}c, H{:=}\{a\}\right)
\end{array}$$

- $\{\partial_{\{c\}}((a+b)c) \xrightarrow{a} \partial_{\{c\}}(c), \partial_{\{c\}}((a+b)c) \xrightarrow{b} \partial_{\{c\}}(c)\}$

$$\dfrac{\dfrac{\dfrac{\overline{b \xrightarrow{b} \surd}}{(a+b) \xrightarrow{b} \surd}}{(a+b)c \xrightarrow{b} c}}{\partial_{\{c\}}((a+b)c) \xrightarrow{b} \partial_{\{c\}}(c)}
\qquad
\begin{array}{l}
\left(\frac{}{v \xrightarrow{v} \surd}, v{:=}b\right) \\
\left(\frac{y \xrightarrow{v} \surd}{x+y \xrightarrow{v} \surd}, x{:=}a, y{:=}b, v{:=}b\right) \\
\left(\frac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, x{:=}(a+b), y{:=}c, v{:=}b\right) \\
\left(\frac{x \xrightarrow{v} x'}{\partial_H(x) \xrightarrow{v} \partial_H(x')} v \notin H, x{:=}(a+b)c, x'{:=}c, v{:=}b, H{:=}\{c\}\right)
\end{array}$$

$$\begin{array}{ll}
\hline a \xrightarrow{a} \surd & (\frac{}{v \xrightarrow{v} \surd}, v{:=}a) \\
\hline (a+b) \xrightarrow{a} \surd & (\frac{x \xrightarrow{v} \surd}{x+y \xrightarrow{v} \surd}, x{:=}a, y{:=}b, v{:=}a) \\
\hline (a+b)c \xrightarrow{a} c & (\frac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, x{:=}(a+b), y{:=}c, v{:=}a) \\
\hline \partial_{\{c\}}((a+b)c) \xrightarrow{a} \partial_{\{c\}}(c) & (\frac{x \xrightarrow{v} x'}{\partial_H(x) \xrightarrow{v} \partial_H(x')} v \notin H, x{:=}(a+b)c, x'{:=}c, v{:=}a, H{:=}\{c\})
\end{array}$$

- $\{\partial_{\{a,b\}}((ab) \parallel (ba)) \xrightarrow{c} \partial_{\{a,b\}}(b \parallel a), \partial_{\{a,b\}}(b \parallel a) \xrightarrow{c} \surd\}$

$$\begin{array}{ll}
ab \xrightarrow{a} b \quad ba \xrightarrow{b} a & (\frac{x \xrightarrow{v} x' \quad y \xrightarrow{w} y'}{x \parallel y \xrightarrow{\gamma(v,w)} x' \parallel y'}, x{:=}ab, y{:=}ba, x'{:=}b, \\
\hline (ab \parallel ba) \xrightarrow{c} (b \parallel a) & y'{:=}a, v{:=}a, w{:=}b) \\
\hline \partial_{\{a,b\}}(ab \parallel ba) \xrightarrow{c} \partial_{\{a,b\}}(b \parallel a) & (\frac{x \xrightarrow{v} x'}{\partial_H(x) \xrightarrow{v} \partial_H(x')} v \notin H, x{:=}(ab \parallel ba), \\
 & x'{:=}(b \parallel a), v{:=}c, H{:=}\{a,b\})
\end{array}$$

$$\begin{array}{ll}
b \xrightarrow{b} \surd \quad a \xrightarrow{a} \surd & \\
\hline (b \parallel a) \xrightarrow{c} \surd & (\frac{x \xrightarrow{v} \surd \; y \xrightarrow{w} \surd}{x \parallel y \xrightarrow{\gamma(v,w)} \surd}, x{:=}b, y{:=}a, v{:=}b, w{:=}a) \\
\hline \partial_{\{a,b\}}(b \parallel a) \xrightarrow{c} \surd & (\frac{x \xrightarrow{v} \surd}{\partial_H(x) \xrightarrow{v} \surd} v \notin H, x{:=}(b \parallel a), v{:=}c, H{:=}\{a,b\})
\end{array}$$

$$\begin{array}{ll}
a \xrightarrow{a} \surd & \\
\hline ab \xrightarrow{a} b & (\frac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, x{:=}a, y{:=}b, v{:=}a)
\end{array}$$

$$\begin{array}{ll}
b \xrightarrow{b} \surd & \\
\hline ba \xrightarrow{b} a & (\frac{x \xrightarrow{v} \surd}{x \cdot y \xrightarrow{v} y}, x{:=}b, y{:=}a, v{:=}b)
\end{array}$$

$$b \xrightarrow{b} \surd \quad (\frac{}{v \xrightarrow{v} \surd}, v{:=}b)$$

$$a \xrightarrow{a} \surd \quad (\frac{}{v \xrightarrow{v} \surd}, v{:=}a)$$

3.4.4 We need to show that the transitions which can be derived from

$$\partial_{\{send(0),send(1),read(0),read(1)\}}((send(0) + send(1)) \parallel (read(0) + read(1)))$$

include both actions *comm*(0) and *comm*(1), and that these are the only transitions. Thus, notice that

$$\partial_{\{send(0),send(1),read(0),read(1)\}}((send(0) + send(1)) \parallel (read(0) + read(1)))$$

can only execute such transitions of the term

$$((send(0) + send(1)) \parallel (read(0) + read(1)))$$

whose labels are not contained in the set

$$\{send(0), send(1), read(0), read(1)\}.$$

Given the communication function

$$\gamma(send(0), read(0)) = comm(0)$$

$$\gamma(send(1), read(1)) = comm(1)$$

the process graph of the term is:

$$\{\partial_{\{send(0),send(1),read(0),read(1)\}}((send(0){+}send(1)) \parallel (read(0){+}read(1))) \xrightarrow{comm(0)} \surd,$$

$$\partial_{\{send(0),send(1),read(0),read(1)\}}((send(0){+}send(1)) \parallel (read(0){+}read(1))) \xrightarrow{comm(1)} \surd\}$$

Let $S = \{send(0), send(1), read(0), read(1)\}$. The derivations for the transitions are:

$$\frac{((send(0)+send(1)) \parallel (read(0)+read(1)) \overset{comm(0)}{\rightarrow} \surd}{\{\partial_S((send(0)+send(1)) \parallel (read(0)+read(1))) \overset{comm(0)}{\rightarrow} \surd} \quad \left(\frac{x \overset{v}{\rightarrow} \surd}{\partial_H(x) \overset{v}{\rightarrow} \surd} v \notin H,\ v{:=}comm(0), H{:=}S,\ x{:=}((send(0)+send(1)) \parallel (read(0)+read(1))\right)$$

$$\frac{((send(0)+send(1)) \parallel (read(0)+read(1)) \overset{comm(1)}{\rightarrow} \surd}{\{\partial_S((send(0)+send(1)) \parallel (read(0)+read(1))) \overset{comm(1)}{\rightarrow} \surd} \quad \left(\frac{x \overset{v}{\rightarrow} \surd}{\partial_H(x) \overset{v}{\rightarrow} \surd} v \notin H,\ v{:=}comm(1), H{:=}S,\ x{:=}((send(0)+send(1)) \parallel (read(0)+read(1))\right)$$

$$\frac{(send(0)+send(1)) \overset{send(0)}{\rightarrow} \surd \quad (read(0)+read(1)) \overset{read(0)}{\rightarrow} \surd}{((send(0)+send(1)) \parallel (read(0)+read(1)) \overset{comm(0)}{\rightarrow} \surd} \quad \left(\frac{x \overset{v}{\rightarrow} \surd\, y \overset{w}{\rightarrow} \surd}{x \parallel y \overset{\gamma(v,w)}{\rightarrow} \surd},\ x{:=}(send(0)+send(1))\ y{:=}(read(0)+read(1))\ v{:=}send(0)\ w{:=}read(0)\right)$$

$$\frac{(send(0)+send(1)) \overset{send(1)}{\rightarrow} \surd \quad (read(0)+read(1)) \overset{read(1)}{\rightarrow} \surd}{((send(0)+send(1)) \parallel (read(0)+read(1)) \overset{comm(1)}{\rightarrow} \surd} \quad \left(\frac{x \overset{v}{\rightarrow} \surd\, y \overset{w}{\rightarrow} \surd}{x \parallel y \overset{\gamma(v,w)}{\rightarrow} \surd},\ x{:=}(send(0)+send(1))\ y{:=}(read(0)+read(1))\ v{:=}send(1)\ w{:=}read(1)\right)$$

$$\frac{read(1) \overset{read(1)}{\rightarrow} \surd}{(read(0)+read(1)) \overset{read(1)}{\rightarrow} \surd} \quad \begin{array}{l}(\frac{}{v \overset{v}{\rightarrow} \surd}, v{:=}read(1)) \\ (\frac{y \overset{v}{\rightarrow} \surd}{x+y \overset{v}{\rightarrow} \surd}, v{:=}read(1), x{:=}read(0), y{:=}read(1))\end{array}$$

$$\frac{send(1) \overset{send(1)}{\rightarrow} \surd}{(send(0)+send(1)) \overset{send(1)}{\rightarrow} \surd} \quad \begin{array}{l}(\frac{}{v \overset{v}{\rightarrow} \surd}, v{:=}send(1)) \\ (\frac{y \overset{v}{\rightarrow} \surd}{x+y \overset{v}{\rightarrow} \surd}, v{:=}send(1), x{:=}send(0), y{:=}send(1))\end{array}$$

$$\frac{read(0) \overset{read(0)}{\rightarrow} \surd}{(read(0)+read(1)) \overset{read(0)}{\rightarrow} \surd} \quad \begin{array}{l}(\frac{}{v \overset{v}{\rightarrow} \surd}, v{:=}read(0)) \\ (\frac{x \overset{v}{\rightarrow} \surd}{x+y \overset{v}{\rightarrow} \surd}, v{:=}read(0), x{:=}read(0), y{:=}read(1))\end{array}$$

$$\frac{send(0) \overset{send(0)}{\rightarrow} \surd}{(send(0)+send(1)) \overset{send(0)}{\rightarrow} \surd} \quad \begin{array}{l}(\frac{}{v \overset{v}{\rightarrow} \surd}, v{:=}send(0)) \\ (\frac{x \overset{v}{\rightarrow} \surd}{x+y \overset{v}{\rightarrow} \surd}, v{:=}send(0), x{:=}send(0), y{:=}send(1))\end{array}$$

3.4.5

- Yes. The process graph of term $\partial_{\{b\}}(ab+c)$ contains action $\partial_{\{b\}}(ab+c) \overset{a}{\rightarrow} \partial_{\{b\}}(b)$ to a deadlocking node $\partial_{\{b\}}(b)$.
- No. The process graph of term $\partial_{\{b\}}(a(b+c))$ is
$$\{\partial_{\{b\}}(a(b+c)) \overset{a}{\rightarrow} \partial_{\{b\}}(b+c), \partial_{\{b\}}(b+c) \overset{c}{\rightarrow} \surd\}$$
where all nodes can execute some action.
- Yes. The process graph of term $\partial_{\{b,c\}}(a(b+c))$ contains action
$$\partial_{\{b,c\}}(a(b+c)) \overset{a}{\rightarrow} \partial_{\{b,c\}}(b+c)$$
to a deadlocking node $\partial_{\{b,c\}}(b+c)$.

- Yes. The process graph of term $\partial_{\{b\}}(ab \parallel c)$ contains a sequence of actions
$$\partial_{\{b\}}(ab \parallel c) \xrightarrow{c} \partial_{\{b\}}(ab) \xrightarrow{a} \partial_{\{b\}}(b)$$
to a deadlocking state $\partial_{\{b\}}(b)$.
- No. The process graph of term $\partial_{\{b,c\}}(ab \parallel c)$ is
$$\{\partial_{\{b,c\}}(ab \parallel c) \xrightarrow{a} \partial_{\{b,c\}}(b \parallel c), \partial_{\{b,c\}}(b \parallel c) \xrightarrow{a} \surd\}$$
where all nodes can execute some action.

B.5.1

- No. Transition system specification
$$T_0 : \emptyset$$
(over signature $\{a\}$) generates empty labelled transition system. Labelled transition system generated by transition system specification
$$T_0 \oplus T_1 : \frac{}{aP}$$
(over signature $\{a, b\}$) is $\{(aP)\}$. Thus, transition (aP) holds w.r.t. $T_0 \oplus T_1$ but does not hold w.r.t. T_0.
- Yes. Transition system specification
$$T_0 : \frac{}{xP}$$
(over signature $\{a\}$) generates transition system $\{(aP)\}$. Labelled transition system generated by transition system specification
$$T_0 \oplus T_1 : \frac{}{xP}, \frac{}{bQ}$$
(over signature $\{a, b\}$) is $\{(bQ), (aP), (bP)\}$. There are no a transitions that hold w.r.t. $T_0 \oplus T_1$ but does not hold w.r.t. T_0.
- No. Transition system specification
$$T_0 : \frac{xQ}{aP}$$
(over signature $\{a\}$) generates empty labelled transition system. Labelled transition system generated by transition system specification
$$T_0 \oplus T_1 : \frac{xQ}{aP}, \frac{}{bQ}$$
(over signature $\{a, b\}$) is $\{(bQ), (aP)\}$. Thus, transition (aP) holds w.r.t. $T_0 \oplus T_1$ but does not hold w.r.t. T_0.
- Yes. Transition system specification
$$T_0 : \frac{xQ}{xP}$$
(over signature $\{a\}$) generates empty labelled transition system, while transition system specification
$$T_0 \oplus T_1 : \frac{xQ}{xP}, \frac{}{bQ}$$
(over signature $\{a, b\}$) generates labelled transition system $\{(bQ), (bP)\}$. There are no a transitions that hold w.r.t. $T_0 \oplus T_1$ but does not hold w.r.t. T_0.
- No. Transition system specification
$$T_0 : \frac{}{x \xrightarrow{c} y}$$
(over signature $\{a\}$) generates labelled transition system
$$\{(a \xrightarrow{c} a)\}.$$
Labelled transition system generated by transition system specification
$$T_0 \oplus T_1 : \frac{}{x \xrightarrow{c} y}$$
(over signature $\{a, b\}$) is
$$\{(a \xrightarrow{c} a), (a \xrightarrow{c} b), (b \xrightarrow{c} a), (b \xrightarrow{c} b)\}.$$
Thus, e.g. transition $(a \xrightarrow{c} b)$ holds w.r.t. $T_0 \oplus T_1$ but does not hold w.r.t. T_0.
- Yes. Transition system specification
$$T_0 : \emptyset$$
(over signature $\{a\}$) generates empty labelled transition system, while transition system specification
$$T_0 \oplus T_1 : \frac{xQ}{xP}, \frac{}{bQ}$$

(over signature $\{a, b\}$) generates labelled transition system $\{(bQ), (bP)\}$. There are no a transitions that hold w.r.t. $T_0 \oplus T_1$ but does not hold w.r.t. T_0.