Laboratory for Theoretical Computer Science T-79.179 Parallel and Distributed Digital Systems Answers for Tutorial 11 April 6th & 7th 2005

- 7.1 By theorem 7.1(g), if each deadlock contains a marked trap in the initial marking, then N has no dead reachable markings. The net has a single deadlock, containing all places of the net. There are three traps, one containing all places, $\{s_1, s_2, s_4, s_5\}$ and $\{s_1, s_3, s_4, s_5\}$. As the traps cover the whole net, it suffices to mark any place in the initial marking to have no reachable dead markings (theorem 7.1(g)). Only an empty initial marking would result in a reachable dead marking, as an empty marking is in this net dead. Initial marking is also always reachable (definition 5.1(b)).
- 7.2 The net is a free-choice net. The net has three deadlocks: $\{s_1, s_3\}, \{s_1, s_2\}$ and $\{s_1, s_2, s_3\}$. However, the only traps are $\{s_1, s_2\}$ and $\{s_1, s_2, s_3\}$. Thus, the deadlock $\{s_1, s_3\}$ does not contain a trap, and thus can not contain a marked trap either. Therefore, no initial marking makes the net live (corollary 7.2(l)).
- 7.3 First, we enumerate all cycles of the net. By corollary 7.3(h), liveness requires at least one marked place on each cycle, while safety requires that each place belongs to a cycle with exactly one marked place. After enumerating all cycles, we can then search for a marking satisfying the abovementioned condition. The cycles are:
 - 1. s_1, s_2
 - 2. s_1, s_{12}
 - 3. s_3, s_4, s_5
 - 4. s_3, s_8
 - 5. $s_3, s_4, s_{10}, s_{11}, s_2$
 - 6. $s_3, s_4, s_{10}, s_{11}, s_{12}$
 - 7. s_4, s_{10}, s_7
 - 8. s_6, s_7, s_8
 - 9. s_6, s_{11}, s_{12}
 - 10. s_6, s_{11}, s_2
 - 11. s_6, s_7, s_4, s_5
 - 12. $s_9, s_{10}, s_{11}, s_{12}$
 - 13. s_9, s_{10}, s_{11}, s_2
 - 14. s_9, s_5
 - 15. s_9, s_{10}, s_7, s_8

Marking the places $s_2, s_3, s_7, s_9, s_{12}$, for example, produce a live and safe initial marking.