T-79.159 Cryptography and Data Security

Lecture 2: Secret Key Cryptography

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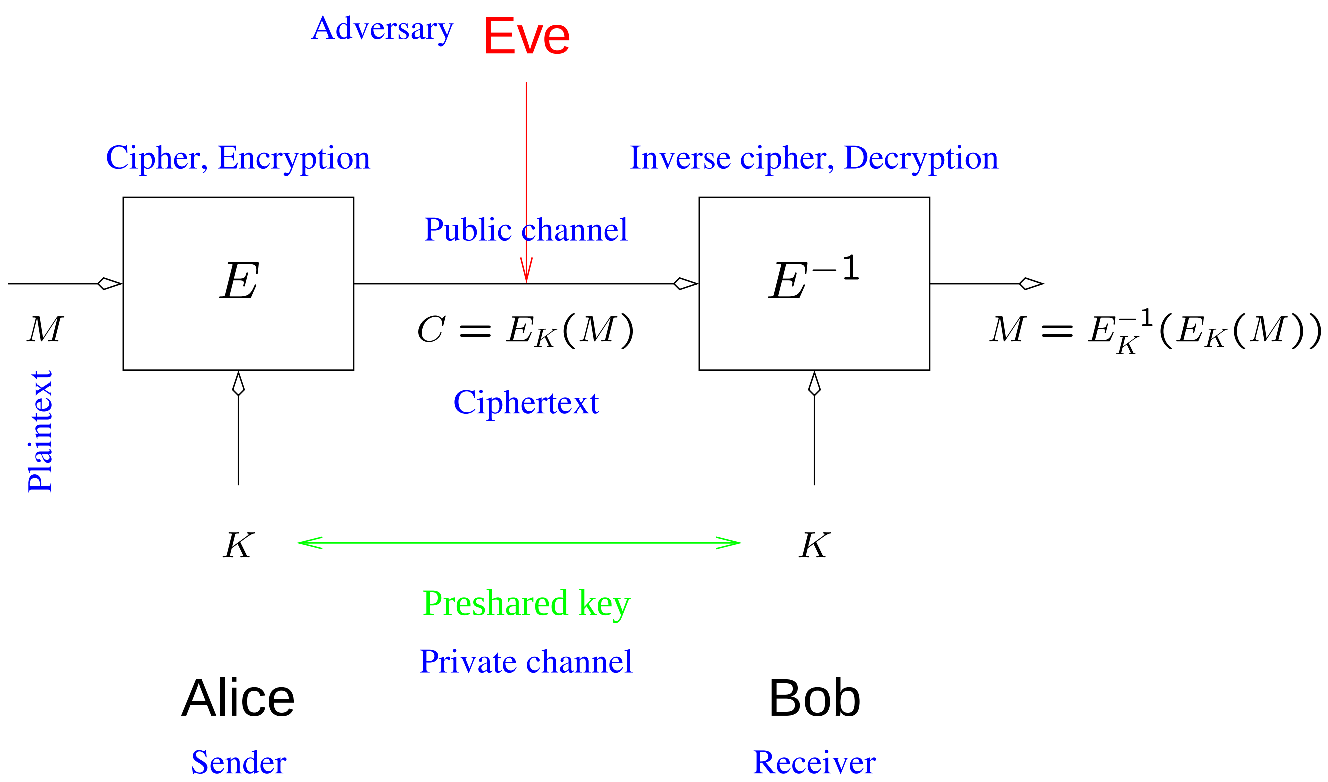
Reminder: Communication Model



Block Ciphers

- A function $E : \mathcal{K} \times \mathcal{P} \to \mathcal{C}$
- $\mathcal{K}$—the key space, $\mathcal{P}$—the plaintext space, $\mathcal{C}$—the ciphertext space
- $E(k, x)$ is often denoted as $E_k(x)$
- E_k is permutation: $(\forall x) E_k^{-1}(E_k(x)) = x$.

Block Ciphers, cont.

- Usually $\mathcal{P} = \mathcal{C} = \{0, 1\}^n$, $\mathcal{K} = \{0, 1\}^k$
- n is the block length, k is the key length
- If k is small, then key can be found by exhaustive search
- If n is small, one can use known-plaintext attack (store all seen plaintext-ciphertext pairs)

Block Ciphers, cont.

- Exhaustively searching k-bit keys takes 2^k time units
- Storing sufficient amount of plaintext-ciphertext pairs takes 2^n memory units
- Birthday attack: $2^{n/2}$ memory units sufficient
- Recommendations: key $k \geq 80$ bits
- Recommendations: block $n \geq 128$ bits

Reminder: Substitution ciphers

- Input and output belong to some set A with $\|A\| = n$
- Key is a permutation π on $(1, 2, 3, \ldots, n)$
- Different "letters" are permuted, according to the key: $A \longrightarrow C$, $B \longrightarrow X$, $C \longrightarrow R$, ...
- Examples: Caesar cipher, shift ciphers, ...

Substitution ciphers, cont.

- There are $2^n!$ permutations
- Storing an arbitrary permutation takes $\log_2(2^n!)$ bits
- By Stirling formula, $x! \approx \sqrt{2\pi x}\left(\frac{x}{e}\right)^x$
- Thus, the key length would be $k = \log_2(2^{128}!)$ bits, or $\approx 2^{134}$ bits, if $n = 128$
- Clearly impractical! (Compare with the lower bound of 80 bits)

Ultimate goal: pseudorandom permutations

- Have a small key of k-bits ($80 \leq k \leq 256$)

- Cipher E should consist of a set of 2^k permutations $\{E_k\}$ out of the total $2^n!$ permutations

- For an attacker who does not know the key, the permutation E_k should look "random"

- That is, deciding whether some permutation π is one of the chosen 2^k permutations should be hard (take $\approx 2^k$ steps)

Permutation ciphers

- Input belongs to A^n for some set A.
- Key is a permutation π on $(1, 2, 3, \ldots, n)$
- Different “letters” are permuted, according to the key.
- Decryption: apply inverse permutation
- Very weak by itself!

Example

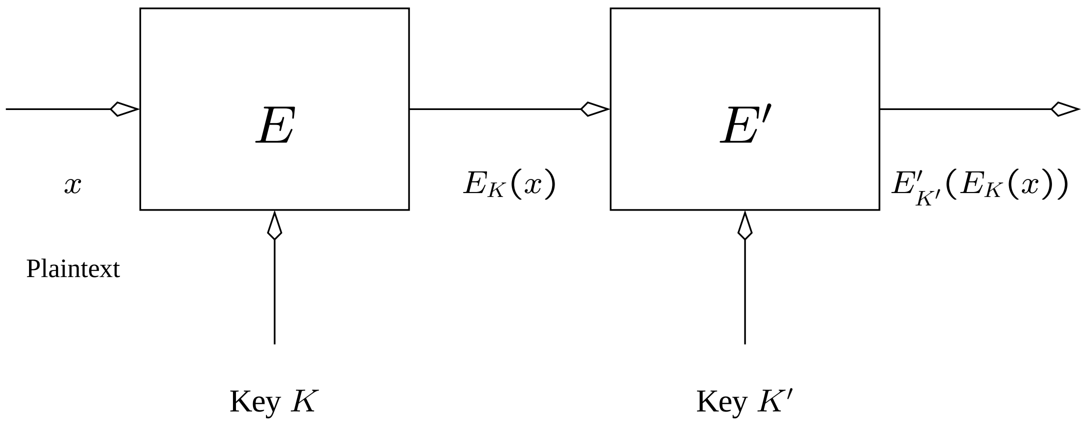
$A = \mathbb{Z}_{26}$, $n = 2$, and $\pi(1) = 2$, $\pi(2) = 1$. A simple example:

`willwehaveabreak`

`iwllewahevbaerka`

Product ciphers

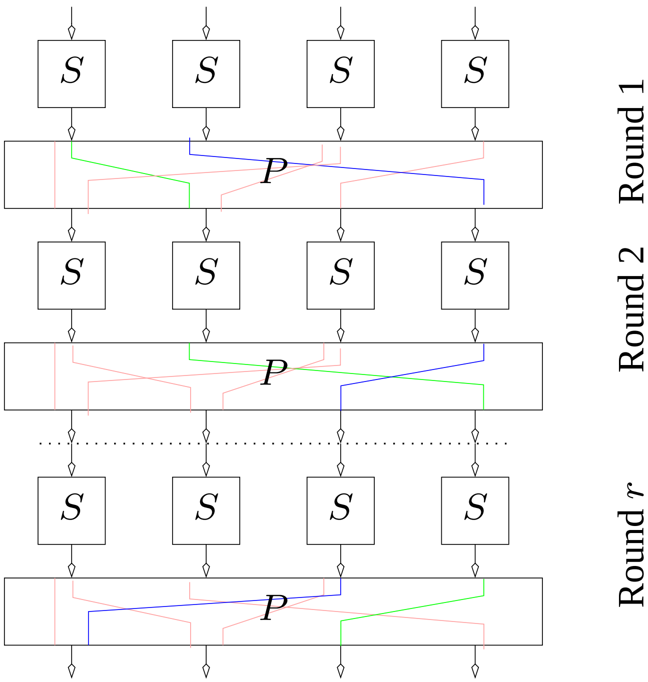
Idea: combine two weak ciphers to get a stronger cipher



Tweak: Use the SAME cipher but with different keys (Question: Why this is not a good idea with the already shown ciphers?)

Tweak II: generate K' from K by using some sophisticated key extension algorithm.

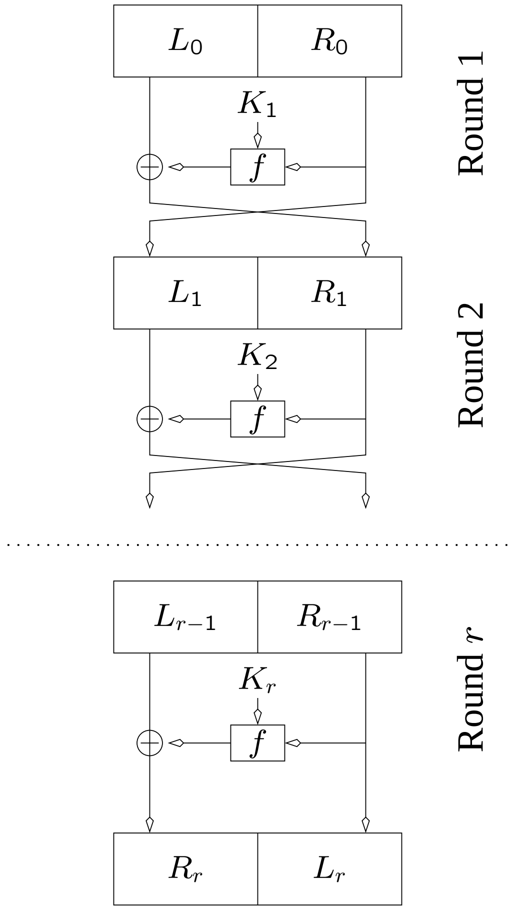
Substitution-Permutation Networks



- Divide the block into small s-bit chunks
- Apply a fixed substitution to every small chunk
- Apply a (key-dependent) permutation to the combined output
- Do this in r rounds
- The bit-permutations mix outputs from different S-boxes
- Some cleverness should be involved to guarantee reversibility

Hybrid: Round = Substitutions + Permutation, and then multiple rounds

Feistel ciphers



f — "suitable" function
K_i — round key
$L_i = R_{i-1}$
$R_i = L_{i-1} \oplus f(K_i, R_{i-1})$
Ciphertext: (R_r, L_r)
Decryption: same
but with the order of round keys reversed

It is *proven* that a Feistel cipher with many rounds is secure if f is a pseudorandom function

DES (1/2)

- In 1973, NBS published a solicitation for a cryptosystems
- One suitable candidate raised: DES (by IBM)
- DES first published in 1975
- Adapted as a standard for “unclassified” communication on January 15, 1977.
- Now superseded by AES

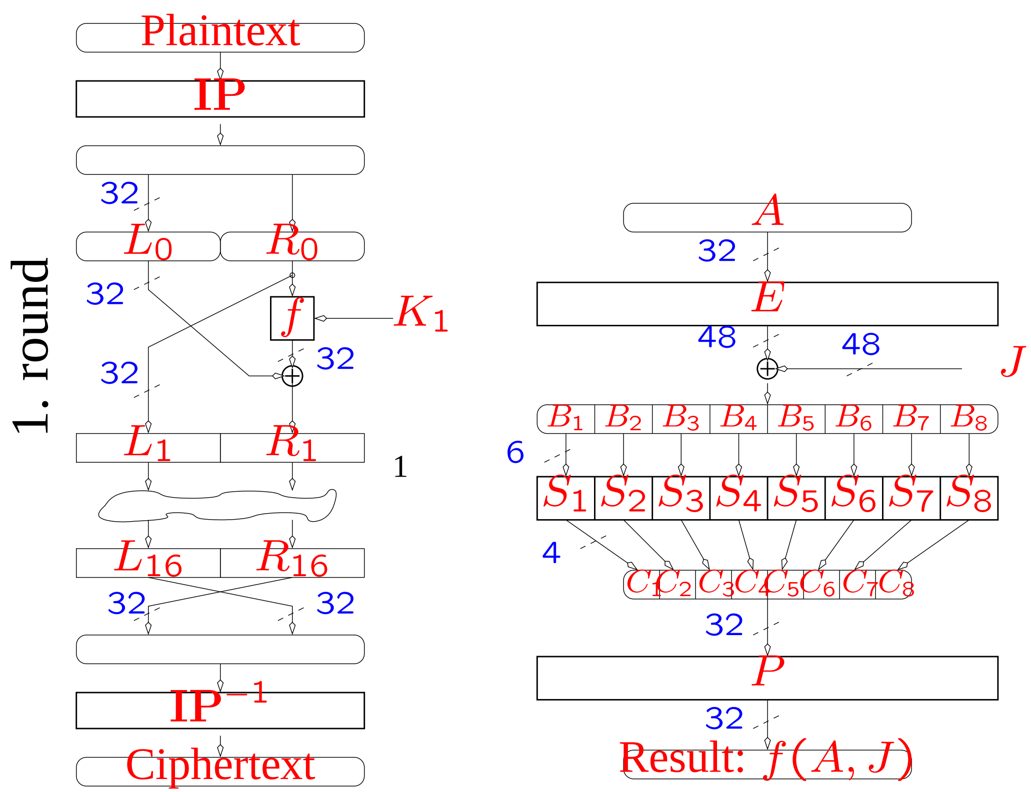
DES (2/2)

- Being the first ever published government-endorsed cryptosystem, DES sparkled a great controversy but also genuine interest
- Wide user-base
- Birth of public cryptanalysis of block ciphers: new methods developed in early 90s to break DES have been used to break many other ciphers
- It seems that DES is essentially secure: best attack requires $\approx 2^{40}$ known plaintext-ciphertext pairs
- Is 2^{40} secure? Is 2^{56} secure?

DES: Description

- A block cipher with 56-bit key, 64-bit block
- Apply a fixed permutation IP to the plaintext x
- Apply a 16-round Feistel cipher to IP(x)
- Apply the inverse permutation IP^{-1}
- Keys K_i are derived from K by using key extension algorithm

DES: Picture



General SchemeFunction $f(A, J)$, where $A = R_i$

DES Components

- $E : \{0,1\}^{32} \longrightarrow \{0,1\}^{48}$: Expansion function. Permutes 32 bits with duplicating half of them

- $S_i : \{0,1\}^6 \longrightarrow \{0,1\}^4$: ith S-box. A nonlinear function

- P: Bit Permutation. Changes bit locations

- Note that E, S_i, P do not depend on the key!

DES: Quick evaluation (1/2)

- Suffers from short key-length: 2^{56} DES operations (for exhaustive search) is currently feasible.

- Key complementation property, $\overline{E_K(x)} = E_{\overline{K}}(\overline{x})$, decreases this to 2^{55}

- ... DES key has been found by using special hardware in 3.5 hours (1999, see http://www.eff.org/descracker/)

DES: Quick evaluation (2/2)

- Best attack: linear cryptanalysis (Matsui 1994, later improved by others), requires $\approx 2^{40}$ known plaintext-ciphertext pairs

- Relatively slow in software: 18 MByte/s on a 800 MHz Pentium

- Very fast in hardware: multi-gigabyte range (designed for hardware)

Differential Cryptanalysis: History

- The first publicly known successful attack against DES (Biham and Shamir, 1990)

- . . . who found DES to be surprisingly strong against the DC

- Don Coppersmith (IBM) later admitted that the designers knew this attack when they designed DES and took it into consideration

Differential Cryptanalysis

- A chosen plaintext attack: n plaintext pairs $(x[i], x^*[i])$, $i \in [1, n]$ are chosen, so that $x[i] \oplus x^*[i] = \Delta x$

- If Δx is well chosen then for some Δy, $E_K(x[i]) \oplus E_K(x^*[i]) = \Delta y$ with a high probability p

- We say that $(\Delta x \to \Delta y)$ has a *differential probability* p

- Use most probable differentials to select some keys as more probable

- Protection: design cipher not to have highly probable differentials

AES

- A competition for the new standard was announced in 1997
- This time, an open competition and 15 candidates participated
- MARS (IBM), RC6 (RSA Labs), Rijndael (Joan Daemen and Vincent Rijmen), Serpent (Anderson, Biham, Knudsen) and Twofish (Counterpane) were selected to the second round
- All five ciphers were found to be sufficiently secure and in late 2000, Rijndael was selected as a winner based on its versatility and clear design principles

AES algorithm (Rijndael): Overview

- Has 128-bit blocks and 128, 192 or 256-bit keys
- The number of rounds depends on the key-length, being 10, 12 or 14
- Specifically designed to be secure against the differential and linear cryptanalysis
- Fast: more than 53 MByte/s on a 800 MHz Pentium
- See http://www.nist.gov/aes for more

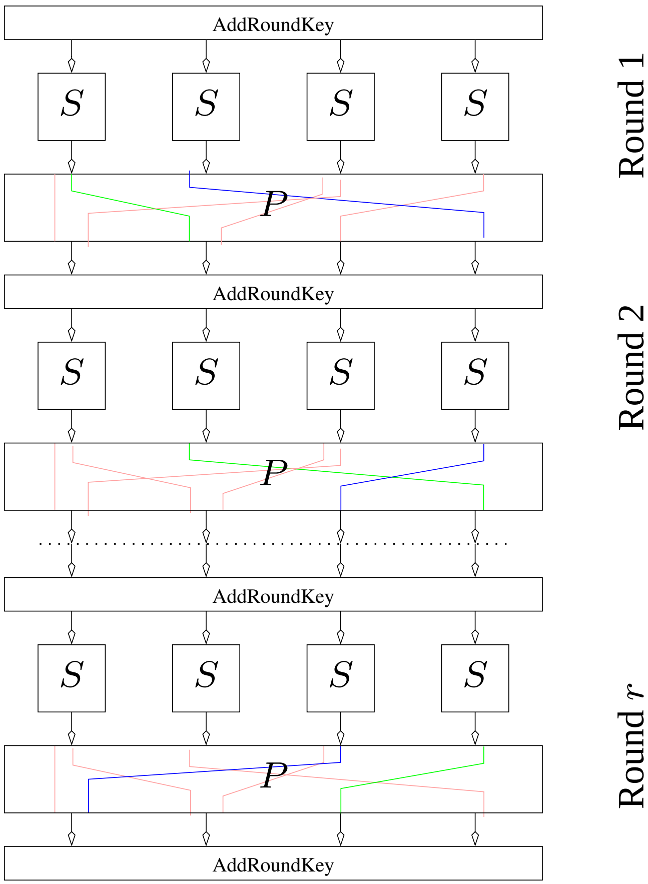
AES: Description

- DES: main operations are XOR, bit permutations and S-boxes (fast in hardware, slow in software)

- AES: main operations are operations in finite field $\mathsf{GF}(2^8)$ and S-boxes (fast in both hardware and software)

- One round consists of the next operations: SubBytes (S-box), ShiftRows, MixColumns (make up the permutation) and AddRoundKey

AES: High Level Overview



- Like general SPN
- AddRoundKey — only dependence on keys
- SubBytes: 8×8 S-box (byte substitution)
- ShiftRow: permutation of bytes
- MixColumns: matrix multiplication of 8-bit finite field elements
- P consists of ShiftRow and MixColumns
- Last row is slightly different
- Decryption has InverseMixColumns (different matrix)

Hybrid: Round = Substitutions + Permutation, and then multiple rounds

One-time pad

0	1	0	1	1	0	0	1	0	1	0	0	1	1	1	0	Plaintext x
																$\oplus$
0	0	1	0	1	1	0	0	0	0	1	1	0	1	0	1	Key k
																$=$
0	1	1	1	0	1	0	1	0	1	1	1	1	0	1	1	Ciphertext y

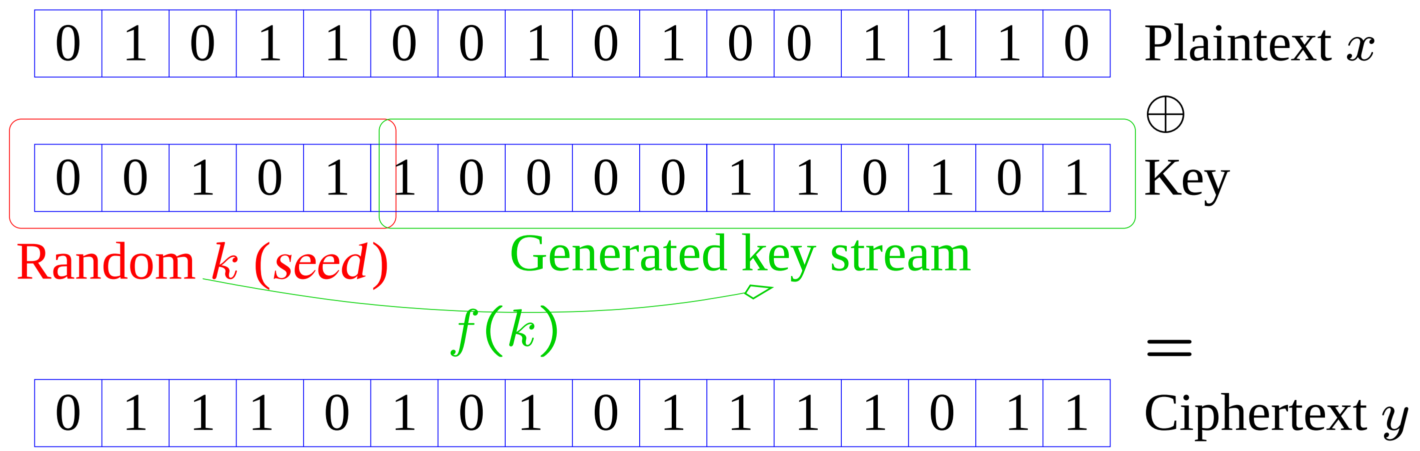
All these key bits are random!

Perfectly secure: if key is random then ciphertext is random. For every key there exists a plaintext that encrypts to this ciphertext. Thus, no information about plaintext is leaked

Bad: *every* perfectly secure cipher requires $|x| = |k| = |y|$. Impractical!

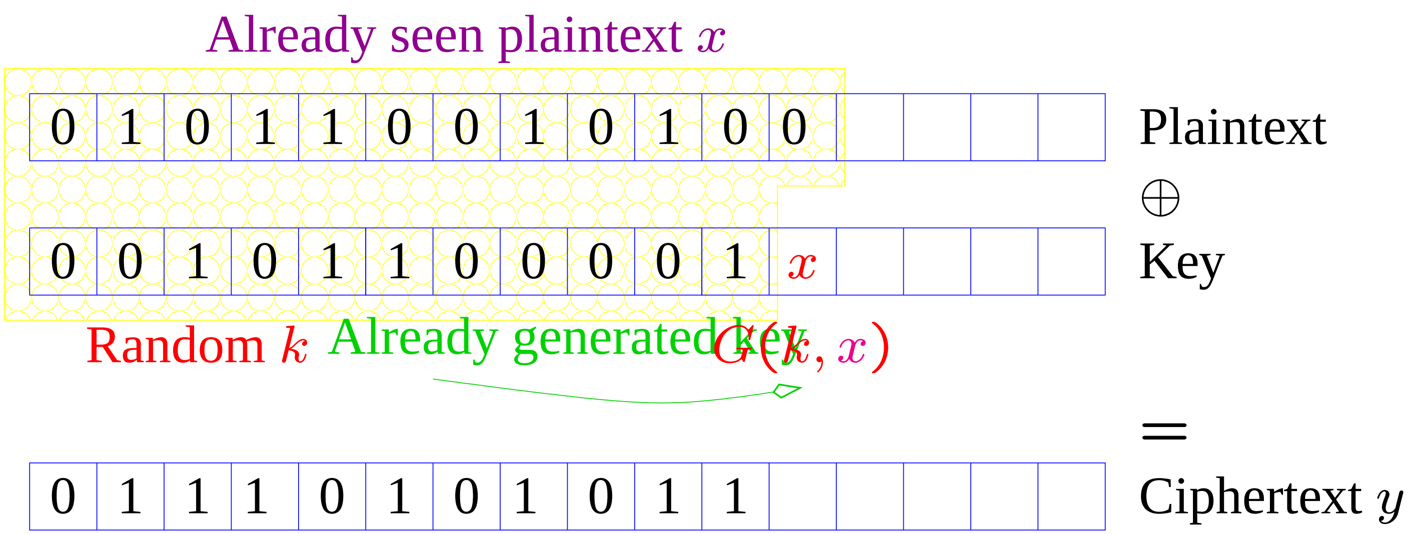
How to improve?

Stream cipher



Idea: generate a long pseudorandom (random-looking) sequence out of the short seed

Stream cipher



That is, key stream might be a function of plaintext.

Stream ciphers: pros

- Do not have to be reversible
  - ⋆ Block ciphers are reversible. This involves increased cost. Stream ciphers are potentially faster

- Intuitively clear what it means for a stream ciphers to be secure: output string is indistinguishable from a random string

- Stream cipher $\approx$ cryptographically strong pseudo-random number generator

Contemporary stream ciphers

- Classical approach, LFSR (Linear Feedback Shift Register), insecure
- Combine two LFSRs by using a well-chosen non-linear function (seen in many ciphers)
- Contemporary ciphers use very different approaches
- While some of stream ciphers are in wide use (RC4, e.g.,), they are far less studied than block ciphers

Contemporary stream ciphers

- RC4: ‘broken” (must discard at least 1024 bytes of the generated key stream), Seal: broken, etc.
- NESSIE project issued a call for stream ciphers. All candidates are broken
- Most efficient attack against the NESSIE candidate LILI128 is by Markku-Juhani Saarinen
- Some secure(?) stream ciphers: Wake, and some new proposals

Why such an situation? (1/2)

- Design philosophy: it's secure if it is not broken!
- The game of cats and mice between cryptographers and cryptanalysts
- . . . Attack, Correct, Attack, Correct, . . .

Why such an situation? (2/2)

- It would be desirable to have a provably secure cipher

- Unfortunately, provably secure ciphers tend

  1. to have a long key: OTP; or

  2. are very slow (public-key cryptosystems are 1000x slower than AES, RC4, . . .)

- Ciphers, provably secure in some situations are very weak in some others