Representing Normal Programs with Clauses

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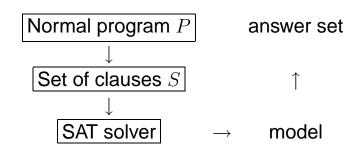
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MOTIVATION

 Our goal is to combine the knowledge representation capabilities of normal logic programs with the efficiency of SAT solvers.



To realize this setting, we present a polynomial and faithful but non-modular translation.

OUTLINE

- ① Terms and Definitions
- ② Characterizing Stability
- **③** Clausal Representation
- ④ Experiments
- 5 Discussion



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① TERMS AND DEFINITIONS

• A rule *r* is an expression of the form

 $h \leftarrow b_1, \ldots, b_n, \sim c_1, \ldots, \sim c_m.$

• We use the following notations for a rule *r*:

$$\begin{split} \mathrm{H}(r) &= \mathsf{h} \quad \text{(head)} \\ \mathrm{B}(r) &= \{\mathsf{b}_1, \dots, \mathsf{b}_n, \sim \mathsf{c}_1, \dots, \sim \mathsf{c}_m\} \quad \text{(body)} \\ \mathrm{B}^+(r) &= \{\mathsf{b}_1, \dots, \mathsf{b}_n\} \\ \mathrm{B}^-(r) &= \{\mathsf{c}_1, \dots, \mathsf{c}_m\} \end{split}$$

We define normal logic programs, or normal programs for short, as sets of rules.



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Syntactic Restrictions

- We distinguish the following special cases:
 - positive rules: $h \leftarrow b_1, \ldots, b_n$
 - atomic rules: $h \leftarrow \sim c_1, \ldots, \sim c_m$
 - strictly unary rules: $h \leftarrow b, \sim c_1, \ldots, \sim c_m$
 - strictly binary rules: $h \leftarrow b_1, b_2, \sim c_1, \ldots, \sim c_m$
- We extend these conditions for sets of rules:
 - positive programs: $\forall r \in P : |B^{-}(r)| = 0$
 - atomic programs: $\forall r \in P : |B^+(r)| = 0$
 - unary programs: $\forall r \in P : |B^+(r)| < 1$
- - binary programs: $\forall r \in P : |B^+(r)| \le 2$



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Least Models

If *P* is a positive normal program, then

- 1. *P* has a unique minimal model, i.e. the least model LM(P) of P;
- 2. $LM(P) = T_P \uparrow \omega = lfp(T_P)$ where the immediately true operator T_P is defined by

 $T_P(A) = \{H(r) \mid r \in P \text{ and } B^+(r) \subseteq A\};$

and

3. $\operatorname{lfp}(\mathbf{T}_P) = \mathbf{T}_P \uparrow i$ for some $i \in \mathbb{N}$, if P is finite.



Level Numbers

Definition: For each atom $b \in LM(P)$, the level number lev(b) of b is the least number n such that $\mathbf{b} \in \mathbf{T}_P \uparrow n - \mathbf{T}_P \uparrow (n-1).$

Example: Consider a positive normal program

 $P = \{r_1 = \mathsf{a} \leftarrow; r_2 = \mathsf{a} \leftarrow \mathsf{b}; r_3 = \mathsf{b} \leftarrow \mathsf{a}\}$

with $LM(P) = \{a, b\}$ and the corresponding leve numbers lev(a) = 1 and lev(b) = 2.



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Stable and Supported Models

Definition: Given an interpretation M, the Gelfond-Lifschitz reduct

 $P^M = \{r^+ \mid r \in P \text{ and } B^-(r) \cap M = \emptyset\}$

where r^+ is defined as $H(r) \leftarrow B^+(r)$ for $r \in P$.

Definition: For a normal program P, an interpretation $M \subseteq \operatorname{At}(P)$ is

1. a stable model of $P \iff M = LM(P^M)$, and

a supported model of
$$P \iff M = T_{P^M}(M)$$
.

Stable and Supported Models

Example: The normal program $P = \{a \leftarrow b; b \leftarrow a\}$ has two supported models $M_1 = \emptyset$ and $M_2 = \{a, b\}$, but only M_1 is stable, as $LM(P^{M_1}) = LM(P) = \emptyset = M_1$ and $LM(P^{M_2}) = LM(P) = \emptyset \neq M_2.$

Some important properties:

- 1. Stable models are also supported models.
- 2. Stable and supported models coincide for atomic programs.
- 3. Clark's completion captures supported models.

② CHARACTERIZING STABILITY

Definition: Let *M* be a supported model of a normal program *P*. A level numbering w.r.t. *M* is a function $\# : M \cup SR(P, M) \rightarrow \mathbb{N}$ such that

- 1. for all $a \in M$, # $a = \min\{\#r \mid r \in SR(P, M) \text{ and } a = H(r)\}$ and
- 2. for all $r \in SR(P, M)$, $\#r = \max\{\#b \mid b \in B^+(r)\} + 1$

where $SR(P, M) = \{r \in P \mid M \models B(r)\}.$

We define $\max \emptyset = 0$ to cover rules r with $B^+(r) = \emptyset$.



Capturing Stable Models

Let M be a supported model of P.

Proposition: If # is a level numbering w.r.t. M, then it is unique.

Theorem: M is a stable model of P

 \iff there is a level numbering # w.r.t. M.



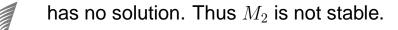
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Capturing Stable Models

Example: Recall the supported models of $P = \{r_1, r_2\}$ with $r_1 = a \leftarrow b$ and $r_2 = b \leftarrow a$: $M_1 = \emptyset$ and $M_2 = \{a, b\}$.

- Since $M_1 \cup SR(P, M_1) = \emptyset$, M_1 is trivially stable.
- For M_2 , the domain $M_2 \cup SR(P, M_2) = M_2 \cup P$ and the set of equations

 $#a = #r_1, #r_1 = #b + 1,$ $#b = #r_2, #r_2 = #a + 1$



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③ CLAUSAL REPRESENTATION

• We use an atomic normal program $Tr_{AT}(P) =$

 $\operatorname{Tr}_{\operatorname{SUPP}}(P) \cup \operatorname{Tr}_{\operatorname{CTR}}(P) \cup \operatorname{Tr}_{\operatorname{MIN}}(P) \cup \operatorname{Tr}_{\operatorname{MAX}}(P)$

as an intermediary representation when translating a normal program P into a set of clauses $Tr_{CL}(Tr_{AT}(P))$.

- Level numbers have to be captured using binary counters which are represented by vectors of propositional atoms.
- Certain primitives have to be represented:
 SEL(c), NXT(c, d), FIX(c), LT(c, d), EQ(c, d).



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- The level numbers associated with rules can be totally omitted, if all non-binary rules r with |B⁺(r)| > 2 are translated away.
- A normal logic program P is partitioned into its strongly connected components C_1, \ldots, C_n on the basis of positive dependencies.
- No counters are needed, if $|H(C_i)| = 1$ holds.
- The number of bits $\nabla C_i = \lceil \log_2(|H(C_i)| + 2) \rceil$ for other strongly connected components C_i .
- Fixed translation schemes can be devised for atomic, strictly unary, and strictly binary rules.

Example

For $P = \{a \leftarrow b; b \leftarrow a\}$, the translation $Tr_{AT}(P)$ contains the following rules for a:

$$\begin{array}{ll} \mathbf{b} \leftarrow \sim \overline{\mathrm{bt}(r_2)}; & \overline{\mathrm{bt}(r_2)} \leftarrow \sim \mathrm{bt}(r_2); & \mathrm{bt}(r_2) \leftarrow \sim \overline{\mathbf{a}}; \\ \overline{\mathbf{a}} \leftarrow \sim \mathbf{a}; & \mathbf{x} \leftarrow \sim \mathbf{x}, \sim \overline{\mathbf{a}}, \sim \min(\mathbf{a}); \\ \mathbf{x} \leftarrow \sim \mathbf{x}, \sim \overline{\mathrm{bt}(r_2)}, \sim \overline{\mathrm{lt}(\mathrm{nxt}(\mathbf{a}), \mathrm{ctr}(\mathbf{b}))_1}; \\ \min(\mathbf{b}) \leftarrow \sim \overline{\mathrm{bt}(r_2)}, \sim \overline{\mathrm{eq}(\mathrm{nxt}(\mathbf{a}), \mathrm{ctr}(\mathbf{b}))} \end{array} \text{ and } \end{array}$$

in addition to four subprograms for choosing the values of ctr(a) and nxt(a) as well as comparing the latter with ctr(b). The rules for b are symmetric.

The only stable model is $N = \{\overline{a}, \overline{b}, \overline{bt(r_1)}, \overline{bt(r_2)}\}.$

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EXPERIMENTS

- We have implemented ${\rm Tr}_{\rm AT}$ and ${\rm Tr}_{\rm CL}$ as respective translators LP2ATOMIC and LP2SAT to be used together with LPARSE.
- Our experiments were run on a 1.67 GHz CPU with 1GB memory.
- In our benchmark, we compute all subgraphs of D_n whose all vertices are mutually reachable.

Here D_n is a directed graph with n vertices and $n^2 - n$ edges: $E_n = \{ \langle i, j \rangle \mid 0 < i \le n, \ 0 < j \le n, \ i \ne j \}.$



Reachability Benchmark

vertex(1n).				
in(V1,V2) :- not out(V1,V2),				
vertex(V1;V2), V1!=V2.				
out(V1,V2) :- not in(V1,V2),				
<pre>vertex(V1;V2), V1!=V2.</pre>				
reach(V,V) := vertex(V).				
reach(V1,V3) :- in(V1,V2), reach(V2,V3),				
vertex(V1;V2;V3), V1!=V2, V1!=V3.				
:- not reach(V1,V2), vertex(V1;V2).				

The order in which the reachability of nodes inferred cannot be determined beforehand.



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Computing All Solutions

Number of Vertices	1	2	3	4	5
SMODELS	0.004	0.003	0.003	0.033	12
CMODELS	0.031	0.030	0.124	293	-
LP2ATOMIC+SMODELS	0.004	0.008	0.013	0.393	353
LP2SAT+CHAFF	0.011	0.009	0.023	1.670	-
LP2SAT+RELSAT	0.004	0.005	0.018	0.657	1879
WF+LP2SAT+RELSAT	0.009	0.013	0.018	0.562	1598
Models	1	1	18	1606	565080
SCCs with $ H(C) > 1$	0	0	3	4	5
Rules (LPARSE)	3	14	39	84	155
Rules (LP2ATOMIC)	3	18	240	664	1920
Clauses (LP2SAT)	4	36	818	2386	7642
Clauses (WF+LP2SAT)	2	10	553	1677	5971

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Computing Only One Solution

Number of Vertices	8	9	10
SMODELS	0.009	0.013	0.022
CMODELS	0.046	0.042	0.055
LP2ATOMIC+SMODELS	>10 ⁴	>10 ⁴	>10 ⁴
LP2SAT+CHAFF	0.771	32.6	254
LP2SAT+RELSAT	2.51	>104	>104
WF+LP2SAT+RELSAT	2.80	4830	>10 ⁴
ASSAT	0.023	0.028	0.037



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5 DISCUSSION

- The new characterization of stable models is based on canonical level numberings.
- \blacksquare The translation function ${\rm Tr}_{\rm AT} \circ {\rm Tr}_{\rm CL}$ has distinctive properties:
 - it covers all finite normal programs *P*,
 - a bijective relationship of models is obtained
 - the Herbrand base At(P) is preserved,
 - the length $||\operatorname{Tr}_{\operatorname{CL}}(\operatorname{Tr}_{\operatorname{AT}}(P))||$ is of order $||P|| \times \log_2 |\operatorname{At}(P)|$, and
 - incremental updating is not needed.



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Conclusions and Future Work

- Various kinds of closures of relations, such as transitive closure, can be properly captured with classical models.
- Our approach is competitive against other SAT-solver-based approaches when the task is to compute all stable models.
- Further optimizations should be pursued for in order to really compete with SMODELS.
- In the future, we intend to study techniques to reduce the number of binary counters and the numbers of bits involved in them.



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