T-79.149 Discrete Structures, Autumn 2004

Tutorial 5, 20 October

1. Verify that the operators corresponding to the combinatorial marking and composition constructions are the same for egf's as for ogf's, i.e. for marking $\hat{c}(z) = zD\hat{a}(z)$ and for composition $\hat{c}(z) = \hat{a}(\hat{b}(z))$.

2. Denote by $b_n^{(r)}$ the number of partitions of the set $[n] = \{1, \ldots, n\}$ where each class contains at most r elements. (Each class must of course by definition be nonempty.) Determine for the sequence $\langle b_n^{(r)} \rangle$ its exponential generating function $\hat{b}^{(r)}(z) = \sum_{n \geq 0} b_n^{(r)} \frac{z^n}{n!}$.

3. Determine the egf's for the classes of permutations where (a) all the cycles are of length three, (b) all the cycles are of even length.