1. Solve the following recurrence equations using the method of generating functions:
   (a) \[
   \begin{aligned}
   &a_0 = 0, \quad a_1 = 1, \\
   &a_n = 5a_{n-1} - 6a_{n-2}, \quad n \geq 2;
   \end{aligned}
   \]
   (b) \[
   \begin{aligned}
   &b_0 = 0, \quad b_1 = 1, \\
   &b_n = 4b_{n-1} - 5b_{n-2}, \quad n \geq 2.
   \end{aligned}
   \]

2. Let \( \langle a_k \rangle = \langle a_0, a_1, a_2, \ldots \rangle \) be a sequence of real numbers, and \( a(x) \) its real-valued generating function (i.e. the formal variable \( x \) is here also considered to be real-valued). Assume that the power series \( \sum_{k \geq 0} a_k x^k \) converges in some neighbourhood of the origin. Which real-number sequences are then represented by the functions \( a'(x) \int_0^x a(t) \, dt \), defined in the same neighbourhood about the origin?

   Use these observations to determine the (real-valued) generating functions for the sequences \( \langle 0, 1, 2, \ldots \rangle \) ja \( \langle 1, \frac{1}{2}, \frac{1}{3}, \ldots \rangle \).

3. The Stirling number of the second kind \( \{n \atop k\} \) indicates in how many ways a set of \( n \) elements can be partitioned into \( k \) nonempty subsets. These numbers satisfy the recurrence equation
   \[
   \{n \atop k\} = \{n-1 \atop k-1\} + k \{n-1 \atop k\} \quad \text{when} \ (n, k) \neq (0, 0); \quad \{0 \atop 0\} = 1.
   \]

   Using this recurrence, construct the generating function \( S_k(z) \) for the sequence \( \langle s_n \rangle \), where \( s_n = \{n \atop k\} \) (i.e. the sequence of Stirling numbers for a fixed value of \( k \)). Derive furthermore from the function \( S_k(z) \) some estimates on the rate of growth of the numbers \( \{n \atop k\} \), as a function of \( n \).