

T-79.149 Discrete Structures, Autumn 2004

Home assignment 3 (due 8 Dec at 4 p.m.)

1. Estimate, up to a precision you consider appropriate, the number of n -bit strings consistent with the regular expression

$$b^*a(b \cup c)^*ab^*$$

(If you are not familiar with the formalism of regular expressions, please consult the course personnel.) Derive your estimate directly from the analytical properties of the respective generating function, without solving its coefficients explicitly.

2. In problem 2 of home assignment 2 it was shown that the exponential generating function of “1-forests” is $\hat{f}(z) = e^{ze^z}$. Estimate, based on this fact and up to a precision you consider appropriate, the number of 1-forests with n nodes. Derive your estimate directly from the analytical properties of the egf $\hat{f}(z)$, without solving its coefficients explicitly.
3. In problem 3 of tutorial 5 exponential generating functions were derived for families of permutations where (a) all the cycles are of length three ($\hat{a}(z) = e^{z^3/3}$) and (b) all the cycles are of even length ($\hat{b}(z) = (1 - z^2)^{-1/2}$). Estimate the number of n -element permutations of types (a) and (b), directly from the respective egf’s without solving their coefficients explicitly.
4. Estimate, using Darboux’s theorem, the number of n -bit strings generated by the context free grammar

$$S \rightarrow aSS \mid bS \mid c$$