T-79.149 Discrete Structures, Autumn 2004

Home assignment 2 (due 17 Nov at 12:15 p.m.)

1. Determine the exponential generating functions of the following combinatorial families:
   (a) Permutations where all the cycles are of odd length.
   (b) Permutations that have at least one cycle of even length.

2. A *forest* is an unordered collection of nonempty unordered rooted labeled trees. (In a "labeled" tree all the nodes are named using e.g. distinct natural numbers. Two trees that have the same structure but a different node labelling are considered different. In an "unordered" tree there is no left-to-right ordering among the subtrees of a given node.) Let us consider forests where all the trees are of height at most one, or *1-forests* for brevity. By listing all the possibilities one can observe that there exist, e.g., three 1-forests with 2 nodes, and ten 1-forests with 3 nodes.
   (a) Determine the exponential generating function counting the number of 1-forests with n nodes.
   (b) Based on the preceding egf, derive a recurrence formula for computing the number of 1-forests with n nodes. Use your formula to compute the exact number of 1-forests with 4 nodes.

3. Determine the number of strings of length n generated by the context free grammar

$$\begin{array}{rcl} S & \rightarrow & a \mid bT \\ T & \rightarrow & S \mid ST \end{array}$$

   (If you are not familiar with the grammar formalism, please consult the course personnel.)

4. Estimate the value of the sum $\sum_{k=1}^{n} k \ln k$ up to order $O(1)$. (*Hint:* Consider first the sum with an upper bound of $n-1$ instead of n.) What estimate can you derive from this for the rate of growth of the product $1^1 \cdot 2^2 \cdots n^n$ as a function of n?