# T-79.149 Discrete Structures (Autumn 2003)

The deadline of Problems 1–4 below is on October 16, 2003, at 17:00.

**Definition used by Problems 3 and 4:** We denote the set of integer numbers by $Z$. A *pseudoneighbour automaton* is a tuple $H = \langle m, k, B, f, g \rangle$ such that $m$ and $k$ are positive integer numbers, $B$ is a finite set of at least two elements, $f$ is a function from $B^k$ to $B$, and $g$ is a function from $Z^m \times \{j \in Z \mid 1 \leq j \leq k\}$ to $Z^m$. The number $m$ is called the *dimension* of $H$. The set of functions from $Z^m$ to $B$ is called the set of *configurations* of $H$. Let $t$ and $u$ be configurations of $H$. Let $p$ be the function from $Z^m$ to $B^k$ such that for each $x \in Z^m$, $\{p(x)\} = \prod_{j=1}^{k} \{t(g(x, j))\}$. We say that $t$ is a *predecessor configuration* of $u$ if and only if for each $x \in Z^m$, $u(x) = f(p(x))$. The *configuration graph* of $H$ is the directed graph such the set of vertices is equal to the set of configurations of $H$, and the set of edges is equal to the set of pairs $\langle v, w \rangle$ such that $v$ is a predecessor configuration of $w$.

## Problems

1. Let us consider finite arrays where the first element is $a$, the last element is $d$, and all other elements belong to $\{b, c\}$. Present a cellular automaton that sorts any given array of this kind into an alphabetically ordered form without ever changing the numbers of occurrences of letters. (6 p)

2. Present a cellular automaton that, given a presentation of the number 1 as the input, enumerates all positive integer numbers in a binary form. Auxiliary columns, auxiliary "colours" and intermediate rows are allowed. (6 p)

3. Show that for each pseudoneighbour automaton, there is some 1-dimensional pseudoneighbour automaton such that the configuration graphs of the automata are isomorphic. (6 p)

4. Let $H = \langle m, k, B, f, g \rangle$ be the pseudoneighbour automaton such that
   $m = 1$, $k = 3$, $B = \{0, 1\}$,
   $f(0,0,0) = f(0,1,1) = f(1,0,1) = f(1,1,1) = 0$,
   $f(0,0,1) = f(0,1,0) = f(1,0,0) = f(1,1,0) = 1$,
   and for each $\langle i, j \rangle \in Z \times \{1, 2, 3\}$, $g(i, j) = i + j - 2$.

   Let $w$ be the configuration of $H$ such that $\{i \in Z \mid w(i) = 1\} = \{0\}$. How many predecessor configurations does $w$ have? Justify your answer. (6 p)