T-79.149 Discrete Structures (Autumn 2003)

The deadline of Problems 1–4 below is on November 27, 2003, at 17:00.

Hypotheses for Problem 1: We assume the definitions given by Sections 2.2, 2.3 and 6.2 in [3], "fixed" as follows.

- Every occurrence of the term "directed graph" is replaced by the term "digraph". (To see why, look at the definition of "graph" in the beginning of Section 2.2.)
- Every set of generators directly defining the arcs or edges in a Cayley graph for a group is inverse-closed. (This is necessary for Theorem 2.3 to hold, even though the classical reference [1] recommended by [3] defines Cayley diagrams without any constraint on the set of generators, except that the group must really be generated by the generators.)
- For any cellular automaton, the graph of the underlying cellular space is a locally finite Cayley graph for some group. (This is taken for granted on page 101, even though Definition 2.4 has a less than perfect correlation with Theorem 2.3, whereas the formulation in the beginning of page 22 is less than perfectly restrictive. For example, the countably infinite set formed by all prime numbers and their inverses is a cardinality-minimal inverse-closed set of generators for the apparently countable multiplication group of positive rational numbers.)
- The set of vertices in the digraph D of Definition 6.1 is finite or countably infinite. Moreover, every vertex has at least one incoming arc. (The latter condition is a necessary condition for the set $Q_i \times \Sigma_i$ in Definition 6.1 to be nonempty, which in turn is a necessary condition for the global dynamics of the random network to be totally defined "without confusion".)
- For any cellular automaton or random network H, let C_H denote the set of configurations of H, and T_H denote the global dynamics of H. For any digraph D, let $V(D)$ denote the set of vertices in D.
- On pp. 100–101, any expression of the form $\langle D, \{X_i\}_i \rangle$ or $\langle D, \{X_i\} \rangle$ is replaced by $\langle D, \{\langle i, X_i \rangle \mid i \in V(D)\} \rangle$. Respectively, $\Pi_i Q_i$ is replaced by $\Pi_{i \in V(D)} Q_i$, and $\cup_i Q_i$ is replaced by $\cup_{i \in V(D)} Q_i$.
- For any cellular space $\langle D, \{\langle i, Q_i \rangle \mid i \in V(D)\}$, the set $\cup_{i \in V(D)} Q_i$ is finite. (A close look at Definition 6.1 reveals that this holds for the underlying cellular space of any random network whenever the above conditions on D hold.)

Motivation for Problem 1: Hybrid cellular automata [4] form a subclass of random networks that has been suggested e.g. for test pattern generation in VLSI design. These automata have cell-dependent rules and are thus not conventional cellular automata. So, Problem 1 is related to construction of a conventional cellular automaton that emulates a given hybrid cellular automaton without using intermediate configurations in the emulation.

Problems

1. Let $\langle \Gamma, \{\langle i, Q_i \rangle \mid i \in V(\Gamma)\} \rangle$ be the underlying cellular space of a random network $J = \langle \Gamma, \{\langle i, L_i \rangle \mid i \in V(\Gamma)\} \rangle$ in such a way that Γ is a Cayley graph for some group (the set of elements in the group apparently being $V(\Gamma)$). Show that there exists a cellular automaton $K = \langle \Gamma, N, M \rangle$ (with the same Γ), a finite set S, and an injection α from C_J to C_K such that $\langle \Gamma, S \rangle$ is the underlying cellular space of K, and $\forall\, x \in C_J : T_K(\alpha(x)) = \alpha(T_J(x))$. (6 p)
2. Describe the benefits of hybrid cellular automata in pseudorandom number generation when compared to conventional cellular automata, using the article [4] as the basis of the description. If you think that there is no relevant benefit, explain why. (6 p)
3. Install DDLab (http://www.ddlab.com/, see also the article [5] presented in the seminar) to some machine where you have an account. Give some concrete examples (not repeating publically available examples) where the installed DDLab computes entropies for one-dimensional and two-dimensional linear cellular automata. (Make sure that the presentation of the examples is detailed enough so that the course staff can trivially repeat them.) Compare those experimental results to the theoretical entropy results of [2] (at least partially familiar from the 3^{rd} set of exercise problems), despite of the fact that the underlying entropy definitions [2, 5] may have somewhat different goals. (6 p)
4. Write an essay on the benefits and weaknesses of DDLab, based on your experience as a user. (6 p)

References

[1] Harold Scott MacDonald Coxeter and William Oscar Jules Moser, *Generators and Relations for Discrete Groups*, Reprint of Fourth Edition, Springer-Verlag, Berlin, Germany, 1980, ix+169 pp. (The book can be found e.g. in the library of Department of Computer Science in University of Helsinki.)

[2] Michele D'amico, Giovanni Manzini, and Luciano Margara, "On computing the entropy of cellular automata," *Theoretical Computer Science*, Vol 290, No. 3, January 2003, pp. 1629–1646. (A PDF version of the article is available e.g. via http://lib.hut.fi/.)

[3] Max Garzon, *Models of Massive Parallelism: Analysis of Cellular Automata and Neural Networks*, Springer-Verlag, Berlin, Germany, 1995, xiv+272 pp.

[4] Peter D. Hortensius, Robert D. McLeod, Werner Pries, D. Michael Miller, and Howard C. Card, "Cellular Automata-Based Pseudorandom Number Generators for Built-In Self-Test," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, Vol. 8, No. 8, August 1989, pp. 842–859. (A PDF version of the article is available e.g. via http://lib.hut.fi/.)

[5] Andrew Wuensche, "Finding Gliders in Cellular Automata," in Andrew Adamatzky (Ed.), *Collision-Based Computing*, Springer-Verlag, Berlin, Germany, 2002, pp. 381–410.