T-79.149 Discrete Structures (Autumn 2003)

The deadline of Problems 1–4 below is on November 6, 2003, at 17:00. See the previous set of exercise problems for the definition of pseudoneighbour automata.

Definition used by Problem 1:
Let N denote the set of nonnegative integer numbers. For any $i \in N$, let $N_{<i} = \{j \in N \mid j < i\}$. Let $D = \{\langle i, j\rangle \mid (i \in N) \wedge (j \in N_{<i+1})\}$. A configuration w of a pseudoneighbour automaton H is *constantly producible* if and only if there exists a function v from D to the set of configurations of H such that
$\forall\, i \in N\ :\ (v(i,i) = w) \wedge$
$\quad (\forall\, j \in N_{<i}\ :\ v(i,j)$ is a predecessor configuration of $v(i,j+1))$.

Problems

1. Let $H = \langle m, k, B, f, g\rangle$ be the pseudoneighbour automaton such that $m = 1$, $k = 2$, $B = \{0,1\}$, $\forall\, \langle b_1, b_2\rangle \in B^2\ :\ f(b_1,b_2) = b_1 b_2$, and $\forall\, \langle i,j\rangle \in Z \times \{1,2\}\ :\ g(i,j) = i + j$. Express the set of constantly producible configurations of H as explicitly as possible. (Make sure that the expression expresses all such configurations and only them.) (6 p)

2. Solve Problem 7 in Garzon's Section 8.7 by assuming that both of the following hold: (i) Equation (4) in Section 2.3 of [1] is equivalent to Garzon's definition of entropy. (ii) In the formulation of Problem 7, "the inequality in Problem 6" refers to the inequality on the last line of Problem 6. (6 p)

3. Using the technique presented in Section 3.1 of [1], compute the entropy for some 1-dimensional linear cellular automaton such that the modulus and the number of nonzero coefficients are of the same magnitude (but not the same) as in Example 2 there. (6 p)

4. Using the technique presented in Section 3.2 of [1], compute the entropy for some 1-dimensional positively expansive cellular automaton such that the number of distinct cycles of the fp-graph and the sum of the lengths of those cycles are of the same magnitude (but not the same) as in Example 3 there. (6 p)

References

[1] Michele D'amico, Giovanni Manzini, and Luciano Margara, "On computing the entropy of cellular automata," *Theoretical Computer Science*, Vol 290, No. 3, January 2003, pp. 1629–1646. A PDF version of the article is available e.g. via http://lib.hut.fi/.