

T-79.149 Discrete Structures, Autumn 2001

Tutorial 9, 28 November

1. Use Darboux's lemma (Theorems 6.8 & 6.9 in the lecture notes) to estimate the coefficients of the following generating functions:

(a) $f(z) = e^{-z/2}\sqrt{1-z}$,

(b) $f(z) = e^{-z+z^2/2}\sqrt{1-z^2}$.

2. The exponential generating function for the class of involutions is $\hat{t}(z) = e^{z+z^2/2}$. (Cf. e.g. home assignment problem 1:4 or p. 28 of the lecture notes.) Use this fact to estimate the number t_n of involutions of n elements.
3. The exponential generating function of the Bell numbers, i.e. the numbers of partitions b_n of n -element sets is $\hat{b}(z) = \exp(e^z - 1)$. (Cf. e.g. tutorial problem 2:3 or p. 27 of the lecture notes.) Use this fact to estimate the size of the numbers b_n .

In case you want to investigate the quality of your estimates in problems 2 and 3, you can easily obtain initial segments of the respective sequences from the "Online Encyclopedia of Integer Sequences" server, <http://www.research.att.com/~njas/sequences/>. Alternatively, you can determine the recurrence formulas for computing the exact values by applying the " zD log trick" to the egf's given in the problems, or recall them from home assignment problem 1:4 and tutorial problem 2:3.