1. As is well known, the ordinary generating function of the Fibonacci numbers is \( f(z) = \frac{z}{1 - z - z^2} \). Derive from this fact an estimate for the size of the Fibonacci numbers \( f_n, n \geq 0 \), based on information about the poles of the function \( f(z) \).

2. The exponential generating function of the Bernoulli numbers is \( \hat{b}(z) = \frac{z}{e^z - 1} \). Derive from this fact an estimate for the size of the numbers \( b_n \). How precise can you make your estimate?

3. Theorem 6.7 of the lecture notes, concerned with estimating the coefficients of meromorphic generating functions, claims that if function \( f(z) = \sum_{n \geq 0} f_n z^n \) has a pole of order \( m \) at \( z_0 \neq 0 \), then its contribution to the coefficient \( f_n \) is

\[
\text{Res}_{z = z_0} \frac{f(z)}{z^{n+1}} = z_0^{-n} P(n),
\]

where \( P(n) \) is a polynomial of degree \( m - 1 \). Prove this claim (i.e. the fact that the residue is of the required form) when (a) \( m = 1 \), (b) \( m \geq 1 \). In the case \( m = 1 \) verify also the explicit formula given for the polynomial (which in this case is just a constant), \( P = \text{Res}(f; z_0)/z_0 \). (Hint: If you wish, you can follow the derivation given in H. Wilf’s book *generatingfunctionology*, page 174.)