Tik-79.148 Introduction to Theoretical Computer Science **Spring 2001**
Practice exam

The exam is split into two parts. To pass the exam you need:

1. At least 5/10 points from the first part.
2. A high enough total score.

Part I Question series A[1]

Write your answers on the separate answering sheet. Remember also to indicate to which question series you are answering to.

1. Let $\Sigma = \{a, b\}$. Which of the following regular expressions define the language: $L = \{w \in \Sigma^* \mid w \text{ has at least one substring } ab\}$

$$\begin{array}{llll} a) & b^*a^*abb^*a^* & c) & (a^*b^*)^*ab(a^*b^*)^* \\ b) & (a \cup b)^*(ab)^*(b \cup a)^* & d) & b^*a^*(ab \cup \emptyset^*)b^*a^* \end{array}$$

2. Which one of the following statements is **not** true?

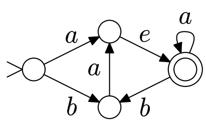
   a) All regular languages are also context-free.

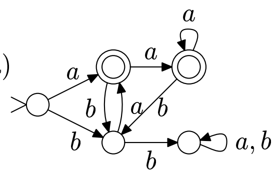
   b) For every μ-recursive function there is a corresponding Turing machine.

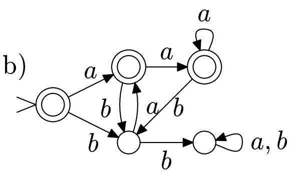
   c) All Turing-acceptable languages are also Turing-decidable.

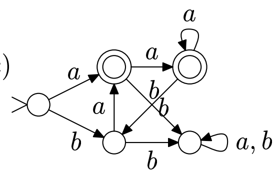
   d) All Turing-decidable languages are also Turing-acceptable.

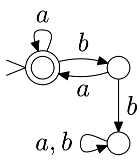
3. Which of the deterministic automata is equivalent with the nondeterministic automaton in the picture?



a) 

b) 

c) 

d) 

4. Which one of the following statements **is** true?

   a) For each type 0 (*unrestricted*) grammar there is a corresponding nondeterministic push-down automaton

   b) The regular languages are undecidable.

   c) The intersection of two regular languages is not necessarily regular.

   d) The intersection of two context-free languages is not necessarily context-free.

5. Which language does the context-free grammar $G = (V, \Sigma, R, S)$ define?

$$V = \{a, b, c, S, A, B\} \qquad \Sigma = \{a, b, c\}$$
$$R = \left\{ \begin{array}{lll} S \to aSb, & S \to A & A \to aA \\ A \to B & B \to bA & B \to e \end{array} \right\}$$

[1]In the real exam there will 10 multiple choice questions. This practice exam has only 5.

a) $L = \{a^n b^n \mid n \geq 0\}$
b) $L = \{a^k (a \cup b)^+ b^k \mid k \geq 0\}$
c) $L = \{(ab)^n a^* b^* \mid n > 0\}$
d) $L = \{a^n (a^* b^*)^* b^n \mid n \geq 0\}$

Part II

1. Construct a deterministic automaton which recognizes the language generated by the regular expression R.

$$R = (bb^*c \cup a)^* \cup (ba^* \cup ca)^*$$

(5p.)

2. Show that, the language L is not regular:

$$L = \{(ab)^n a^k \mid n > k, k \geq 0\}$$

(5p.)

3. Construct a context-free grammar which generates the language:

$$L = \{a^n b^m c^k \mid k = |n - m|\}$$

What is the corresponding pushdown automaton?

(5p.)

4. a) Show that, the concatenation of two context-free languages is context-free. (2p.)
   b) Show, on a general level, the operating principle of the universal Turing machine. (3p.)