

**T-79.148 Introduction to Theoretical Computer Science (2 cr)**  
**Exam Mon 20 Dec 2004, 4–7 p.m.**

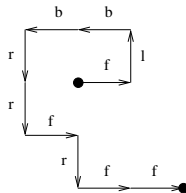
Write down on each answer sheet:

- Your name, department, and study book number
- The text: “T-79.148 Introduction to Theoretical Computer Science 20.12.2004”
- The total number of answer sheets you are submitting for grading

1. Show that the following languages are regular:

- (a)  $\{w \in \{0,1\}^* \mid w \text{ contains three consequent zeros or three consequent ones (or both)}\}$ ; 5p.
- (b)  $\{w \in \{0,1\}^* \mid w \text{ contains neither three consequent zeros nor three consequent ones}\}$ ; 5p.
- (c)  $\{w \in \{0,1\}^* \mid \text{the number of ones in } w \text{ is a multiple of three (possibly zero)}\}$ ; 5p.
- (d)  $\{w \in \{0,1\}^* \mid |w| \geq 3 \text{ and the third-to-last symbol in } w \text{ is a } 1\}$ . 5p.

2. A *party walk* is a sequence of consequent steps, whose direction with respect to the starting point is either forward (*f*), backward (*b*), left (*l*) or right (*r*). For instance, the sequence *flbrrrfrff* describes the following walk, whose total result is to move the walker a distance of two steps forward (and concurrently two steps to the right):



Design a context-free grammar that generates all party walks whose total result is to move the walker at least one step forward from the starting point (ignoring any possible sideways movement). 10p.

3. (a) Design a nondeterministic pushdown automaton that recognises the language

$$L = \{w \in \{0,1\}^* \mid |w| \text{ is odd and the middle symbol of } w \text{ is a } 1\}.$$

8p.

(b) Prove (precisely!) that the language discussed in part (a) can not be recognised by a finite automaton. 7p.

4. (a) Prove that if the languages  $L \subseteq \{0,1,\#\}^*$  and  $L' \subseteq \{0,1\}^*$  are context-free, then so is the language  $L'' = L[L'] \subseteq \{0,1\}^*$ , whose words are obtained from the words in  $L$  by replacing each  $\#$ -symbol by some word in  $L'$  (not necessarily always the same). 8p.

(b) The same problem as in part (a), but with respect to recursively enumerable (i.e. recognisable, semidecidable) rather than context-free languages. 7p.

Total 60p.