Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Sat 23 Oct 2004, 10 a.m. – 1 p.m.

Write down on each answer sheet:
- Your name, department, and student number
- The text: "T-79.148 Introduction to Theoretical Computer Science 23.10.2004"
- The total number of answer sheets you are submitting for grading

1. Regular languages.

   (a) Let

   $$L = \{w \mid w = a_1 a_2 \ldots a_n, n \geq 0, a_i \in \{0,1\}, a_1 = a_3 = a_5 = \ldots = 1\}.$$

   In other words, L consists of those binary strings that have 1s in odd-numbered positions. Show that L is regular. *7 p.*

   (b) Let

   $$L = \{w \mid w \in \{0,1\}^*, w \text{ contains an even number of 0s or exactly two 1s.}\}.$$

   Show that L is regular. *8 p.*

2. A context-free language.

   Let us examine the language $L = \{a^m b^n \mid m < n\}$.

   (a) Design a context-free grammar that generates the language L. *8 p.*

   (b) Show that L is not regular. *7 p.*

3. A computable function.

   Design a Turing machine that computes the function $f(n) = n \bmod 3$. The machine receives the string 1^n as input, and at the end of the computation it must leave $n \bmod 3$ ones at the beginning of the tape, where $n \bmod 3$ denotes the remainder when n is divided by 3. The other ones must be overwritten with the symbol #.

   (a) Present your Turing machine as a state diagram. *6 p.*

   (b) Write an overview of the method your machine uses. *6 p.*

   (c) Give the computation of your machine with the inputs 111 and 11111. *3 p.*

4. Solvable problems.

   (a) Describe, on a general level, an algorithm for deciding whether a given regular expression r defines the language $L(r) = \Sigma^*$ for some alphabet Σ. *8 p.*

   (b) Describe, on a general level, an algorithm for deciding whether two given regular expressions r and s define the same language, i.e., whether $L(r) = L(s)$. *7 p.*

   Hint: you should probably look at the automaton representations of the languages.

Total 60 p.