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T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Mon 23 Aug 2004, noon − 3 p.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 23.8.2004"
- The total number of answer sheets you are submitting for grading

1. Let L be the language that the regular expression $(ab \mid aba)^*$ defines.

   (a) Design a nondeterministic finite state automaton that recognizes L.   *7 p.*

   (b) Find the minimal deterministic finite state automaton that recognizes L.   *8 p.*

2. Consider the following context free grammar G.

$$
\begin{aligned}
S &\ \rightarrow\ s \mid T \\
T &\ \rightarrow\ \textbf{if } C \textbf{ then } S \mid\ \textbf{if } C \textbf{ then } S \textbf{ else } S \\
C &\ \rightarrow\ c
\end{aligned}
$$

   (a) Give the parse tree of the string "**if** c **then if** c **then** s **else** s" for G.   *7 p.*

   (b) Show that G is ambiguous.   *8 p.*

3. Design a deterministic single-tape Turing machine that adds one to the binary number it receives as input. For example, the machine should replace input string 1001 by the string 1010 and the input string 11 by the string 100. Give a description of your algorithm, present the Turing machine as a state chart and give the computations of the machine with the input strings 1011 and 111.   *15p.*

4. A language class C is closed under complement, if for every $L \in C$ also $\bar{L} \in C$.

   (a) Show that the class of regular languages is closed under complement.   *5 p.*

   (b) Show that the class of context-free languages is not closed under complement.
       Hint: The language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free.   *5 p.*

   (c) Show that the class of recursive languages is closed under complement.   *5 p.*

Total 60 p.