1. Let the alphabet of the finite state automaton $M$ be $\Sigma = \{a, b\}$. The transition function of $M$ is described in Figure 1: the initial state is marked with $\rightarrow$ and accepting final states are marked with $\leftarrow$. The automaton $M$ recognizes the language $L$.

(a) Determine the minimal deterministic finite state automaton that recognizes the language $L$. 9 p.
(b) Present $L$ as a regular expression. 6 p.

![Figure 1: The finite state automaton $M$ in tabular form](image)

2. Let us define a string of properly nested parentheses inductively: $\varepsilon$ is a string of properly nested parenthesis, and if $x$ and $y$ are strings of properly nested parenthesis, then so are $(x)$, $[y]$, and $xy$. For example, $([])$, $[([])]$ are strings of properly nested parenthesis, but $(])$, $[)$ and $][]$ are not. Let $L$ be the language of strings of properly nested parenthesis.

(a) Prove in detail that $L$ is not regular. 8 p.
(b) Design a context-free grammar that produces $L$. 8 p.
(c) Design a pushdown automaton that recognizes $L$. 9 p.

3. (a) Define the concepts recursive language and recursively enumerable language. What is their most important difference? 5 p.
(b) Prove that if the language $L$ is recursive, then so is the language

$$L^* = \bigcup_{k \geq 0} L^k = \{w_1 \ldots w_k \mid k \geq 0, w_i \in L \text{ for all } 1 \leq i \leq k\}.$$

15 p.

Total 60 p.