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Laboratory for Theoretical Computer Science
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T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Fri 19 Dec 2003, 10 a.m. – 1 p.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 19.12.2003"
- The total number of answer sheets you are submitting for grading

1. Describe the following languages **both** in terms of regular expressions **and** in terms of deterministic finite automata:

   (a) $L = \{w \in \{a,b,c\}^* \mid w \text{ contains } ac \text{ or } abc \text{ (or both) as a substring}\}$ *7p.*

   (b) $\bar{L} = \{w \in \{a,b,c\}^* \mid w \text{ does not contain } ac \text{ or } abc \text{ as a substring}\}$. *8p.*

   Hint: It may be easiest to derive the solution to (b) from the solution to (a).

2. (a) Describe the language generated by the following context-free grammar:

$$\begin{aligned} S &\rightarrow ASb \mid \epsilon \\ A &\rightarrow aA \mid a \end{aligned}$$

   5 p.

   (b) Show that the grammar in (a) is ambiguous. *5 p.*

   (c) Give an unambiguous context-free grammar that generates the same language as the grammar in (a). *5 p.*

3. Design a deterministic single-tape Turing machine that replaces its input string $w \in \{a,b\}^*$ by the string ww. (Present the Turing machine preferably as a state diagram rather than a transition table.) Show the computation sequences ("runs") of your machine on inputs a and ab. *15p.*

4. (a) Show that if the language L is regular, then the language $\bar{L}$ (the complement of L) is also regular. *5p.*

   (b) Show that if the languages L_1 and L_2 are context-free, then the language $L_1 \cup L_2$ is also context-free. *5p.*

   (c) Show that if the languages L_1 and L_2 are recursively enumerable, then the language $L = L_1 \cap L_2$ is also recursively enumerable. *5p.*

Total 60p.

After the exam, please fill in the feedback form on the WWW page of the course.