

**Helsinki University of Technology**  
**Laboratory for Theoretical Computer Science**  
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**T-79.148 Introduction to Theoretical Computer Science (2 cr)**  
**Exam Wed 7 May 2003, 9–12 a.m.**

Write down on each answer sheet:

- Your name, department, and study book number
- The text: “T-79.148 Introduction to Theoretical Computer Science 7.5.2003”
- The total number of answer sheets you are submitting for grading

1. Describe the following languages **both** in terms of regular expressions **and** in terms of deterministic finite automata:

- (a)  $\{w \in \{0, 1\}^* \mid w \text{ contains an even number (possibly zero) of 1's}\}$  4p.
- (b)  $\{w \in \{0, 1\}^* \mid w \text{ contains an odd number of 1's}\}$  4p.
- (c)  $\{w \in \{0, 1\}^* \mid w \text{ contains an even number (possibly zero) of 1's and ends in a 0, or contains an odd number of 1's and ends in a 1}\}$ . 7p.

(*Hint:* In part (c) it may, depending on your solution method, be useful to first design a nondeterministic automaton.)

2. (a) Design a context-free grammar for the language

$$L = \{a^n b^m \mid n \geq 0 \text{ and } m = n \text{ or } m = 2n\}.$$

Draw the corresponding parse tree for the sentence  $aabbbb$ . 8p.

(b) Prove (precisely!) that the language  $L$  in part (a) is not regular. 7p.

3. Design a (nondeterministic) pushdown automaton that recognises (accepts) the language  $L$  considered in problem 2. (Present the automaton preferably as a state diagram rather than a transition table.) Show the accepting computation sequences (“runs”) of your automaton on the inputs  $ab$  and  $abb$ , and explain why it doesn’t accept the input  $aabbb$ . 15p.

4. *One* of the following:

(a) Show that if the language  $L$  is context-free then so are the following languages:

$$L^* = \bigcup_{k \geq 0} L^k = \{w_1 \dots w_k \mid k \geq 0, w_i \in L \text{ for all } i = 1, \dots, k\},$$
$$L^R = \{w^R \mid w \in L\}.$$

(Here  $w^R$  denotes the *reversal* of string  $w$ , i.e.  $w$  written backwards.) 15p.

(b) Explain what is meant by a universal Turing machine, and outline at the level of basic operating principles the construction of such a machine. 15p.

*Total 60p.*