Helsinki University of Technology
Laboratory for Theoretical Computer Science
Pekka Orponen, tel. (451)5246

T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Mon 28 Oct 2002, 12–3 p.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 28.10.2002"

1. (a) Give a regular expression that describes the language

$$\{w \in \{0,1\}^* \mid w \text{ is of even length and ends in a } 0, \text{ or } w \text{ is of odd length and ends in a } 1.\}$$

3p.

   (b) Design a deterministic finite automaton that recognises the language in part (a). (*Hint:* Design first a nondeterministic automaton.) *4p.*

2. (a) Prove (precisely!) that the language

$$L = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$$

   is not regular. *3 p.*

   (b) Design a context-free grammar for the language L in part (a). *3 p.*

   (c) Show that the grammar you gave in part (b) is ambiguous. *3 p.*

3. Design a nondeterministic single-tape Turing machine that recognises ("decides") the language L considered in problem 2. (Present the Turing machine in terms of state or machine diagrams, rather than transition tables.) Show all the computation sequences ("runs") of your machine on inputs *abbcc* and *abbc*. *7p.*

4. *One* of the following:

   (a) Prove that all regular languages are context-free, without appealing to the correspondence between context-free grammars and pushdown automata. (Using this correspondence would make the proof trivial, since finite state automata are a special case of pushdown automata.) Illustrate your proof with an example. *7p.*

   (b) Prove directly, without appealing to Rice's theorem, that it is an undecidable problem to test whether two Turing machines, given as input, recognise exactly the same language. *7p.*

Total 30 p.