Helsinki University of Technology
Laboratory for Theoretical Computer Science
Pekka Orponen, tel. (451)5246

T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Mon 12 Aug 2002, 12–3 p.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 8.5.2002"

This exam is in two parts. To pass the exam, you need to achieve:
1. A minimum of 5/10 points on Part I, which contains 5 problems at 1–4 points each.
2. A sufficiently high sum total of points altogether.

Part I
For the problems in this Part, write down the answer sequences on the separate answer sheet provided. **Remember to write down your identification data also on this sheet,** but don't use it for any other work. Remember to return the answer sheet together with your solutions to Part II of the exam!

1. With respect to the following claims, list in order whether each one of them is *true (T)* or *false (F)*. (Each correct answer is worth $\frac{1}{2}$ points.) *4 points*

   (a) Every language described by a regular expression can be recognised by a deterministic finite automaton.

   (b) The union of any two context-free languages is context-free.

   (c) The intersection of any two context-free languages is regular.

   (d) Deterministic pushdown automata can recognise some nonregular languages.

   (e) Every context-free language can be recognised ("accepted", "semidecided") by a deterministic Turing machine.

   (f) Every recursively enumerable ("Turing-recognisable", "semidecidable") language can be generated by a context-free grammar.

   (g) The halting problem for Turing machines is decidable.

   (h) The complement of any recursively enumerable language is also recursively enumerable.

2. Which one of the languages described below does the following context-free grammar generate:
$$S \to aSb \mid bSa \mid aaSb \mid bSaa \mid SS \mid a \mid b \mid \varepsilon?$$

   (a) $(a \cup b)^*$

   (b) $\{w \in \{a,b\}^* \mid w \text{ contains at least as many } a\text{'s as } b\text{'s}\}$

   (c) $\{w \in \{a,b\}^* \mid w \text{ contains at least as many } a\text{'s as } b\text{'s, and at most twice as many}\}$ *1 point*

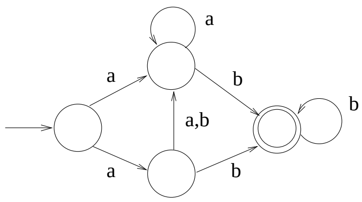
3. Which one of the following regular expressions describes the language

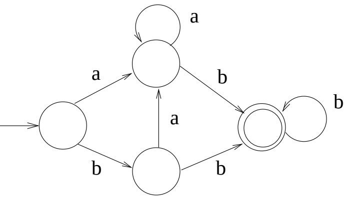
$$L = \{w \in \{0,1\}^* \mid w \text{ does not contain the substring } 111\}?$$

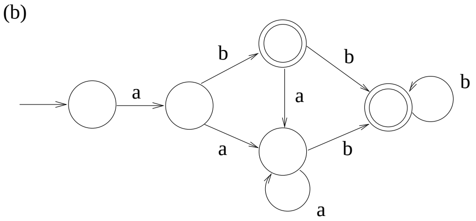
(a) $(\varepsilon \cup 1 \cup 11)(00^*(\varepsilon \cup 1 \cup 11))^*$
(b) $(0^*(10 \cup 110 \cup 01 \cup 011)0^*)^*$
(c) $(0^*(1 \cup 11)0)^*(\varepsilon \cup 1 \cup 11)$
(d) $(0^* \cup 10^* \cup 110^*)^*(\varepsilon \cup 1 \cup 11)$

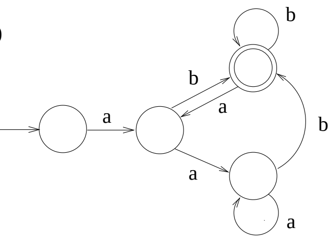
1 point

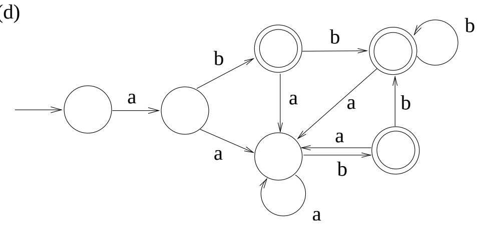
4. Which one of the deterministic finite automata given below is equivalent (i.e. recognises the same language as) the following nondeterministic automaton?



(a)


(b)


(c)


(d)


2 points

5. Consider the context-free grammar $G = \{\, S \to ASb \mid AS \mid \varepsilon, \quad A \to aA \mid \varepsilon \,\}$. With respect to the following claims, list in order whether each one of them is *true (T)* or *false (F)*. (Each correct answer is worth $\frac{1}{2}$ points.) *2 points*

(a) The string $ababb$ is generated by the grammar G.
(b) Every string of the form $a^m b^n$, $m \geq n \geq 0$, is generated by G.
(c) The grammar G is ambiguous.
(d) The language generated by G is regular.

Total 10 points

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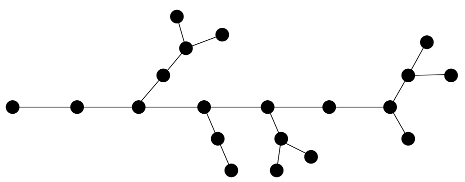
Part II

1. (a) Design a deterministic finite automaton that determines whether a given binary string contains at least two possibly overlapping occurrences of the substring 11. Thus, the automaton should e.g. accept the strings 01100110 and 00111, and reject the strings 11 and 1101. *3p.*

   (b) Give a regular expression describing the language recognised by the automaton in part (a). *2p.*

2. Is the language
$$L = \{0^m 1^n \mid m \leq n\}$$
(a) regular, (b) context-free but nonregular, (c) recursive but not context-free, (d) nonrecursive? Give a *precise justification* for your answer. (Unjustified answer 0 points.) *5p.*

3. A *fern* consists of a stem and a number of subferns rooted on the left and right sides of the stem. For instance, the following structure is a fern:



A fern structure can be described by a string where each unit of the stem is denoted by a letter s, and each subfern is described by a similar string in parentheses, located at the point where the subfern is rooted, and prefixed by l or r depending on whether the subfern occurs on the left or right side of the main stem, respectively. For instance, the string representation corresponding to the above example would be:
$$ssl(ssl(s)r(s))sr(ss)sr(sl(s)r(s))ssl(sr(s)s)r(s).$$
Design a context-free grammar describing the structure of ferns. (I.e. the grammar should generate all and only the valid fern strings.) *5p.*

4. Let $A \subseteq \Sigma^*$ be some set of strings. Show that if both A and its complement $\bar{A} = \Sigma^* - A$ are recursively enumerable ("Turing-recognisable", "semidecidable"), then A is recursive ("decidable"). Deduce from this result the fact that the set
$$\{\ P\#x \mid \text{C-program } P \text{ goes to an infinite loop on input } x\ \}$$
is not recursively enumerable. *5p.*

Each problem 5 p., total 20 p.

Both parts combined total 30 p.