Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Wed 8 May 2002, 9–12 a.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 8.5.2002"

This exam is in two parts. To pass the exam, you need to achieve:
1. A minimum of 6/10 points on Part I, which contains 5 problems at 1–4 points each.
2. A sufficiently high sum total of points altogether.

Part I

For the problems in this Part, write down the answer sequences on the separate answer sheet provided. **Remember to write down your identification data also on this sheet,** but don't use it for any other work. Remember to return the answer sheet together with your solutions to Part II of the exam!

1. With respect to the following claims, list in order whether each one of them is *true (T)* or *false (F)*. (Each correct answer is worth $\frac{1}{2}$ points.) *4 points*

   (a) The union of any two regular languages is context-free.

   (b) Every language that can be recognised by a nondeterministic pushdown automaton can be generated by a context-free grammar.

   (c) Every language that can be recognised by a deterministic pushdown automaton can be described by a regular expression.

   (d) There exist nonrecursive (i.e. "undecidable") context-free languages.

   (e) Nondeterministic Turing machines recognise ("accept", "semidecide") exactly the recursively enumerable languages.

   (f) The language $\{a^n b^n \mid n \geq 0\}$ can be recognised on a nondeterministic finite automaton.

   (g) The complement of every recursive ("decidable") language is recursively enumerable ("Turing-recognisable", "semidecidable").

   (h) The computation of a deterministic Turing machine terminates on every input.

2. Classify the following language in terms of the Chomsky hierarchy, using the following notations: 3 = regular, 2 = context-free and not regular, (1 not in use), 0 = recursively enumerable and not context-free, X = not recursively enumerable. (Each correct answer is worth $\frac{1}{2}$ points.) *2 points*

   (a) $\{w \in \{a,b\}^* \mid w = w^R\}$

(b) $\{cw \mid \text{The Turing machine } M_c \text{ encoded by the string } c \text{ halts on input } w\}$
(c) $\{a^{2n}b^{2m} \mid n, m \geq 0\}$
(d) $\{cw \mid \text{The Turing machine } M_c \text{ encoded by the string } c \text{ rejects input } w\}$

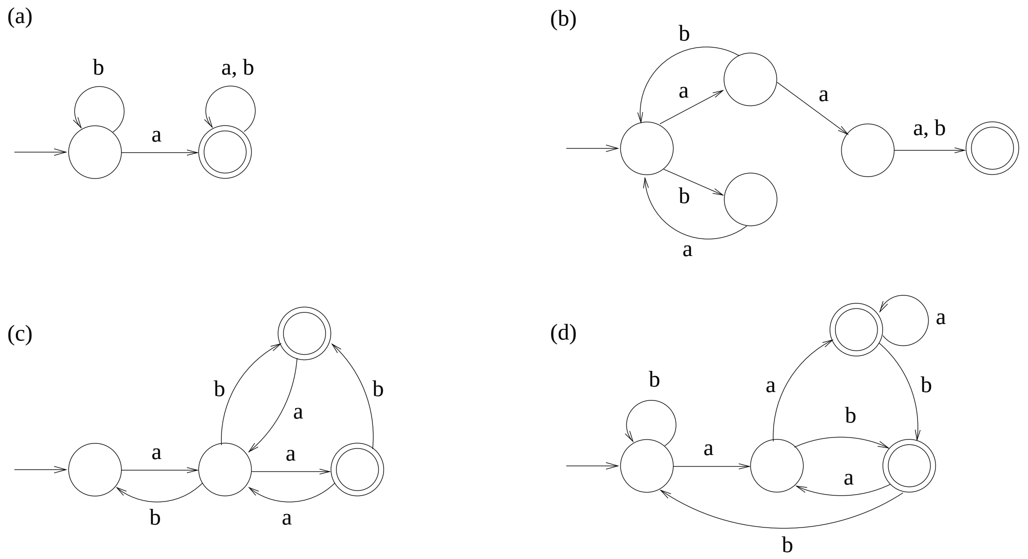
3. Which one of the following regular expressions describes the language

$$L = \{w \in \{0,1\}^* \mid w \text{ contains equally many 0's and 1's (mod 3)}\} :$$

(a) $((0 \cup 1)^*(00 \cup 11)^*(000 \cup 111)^*)^*$
(b) $(01 \cup (00 \cup 1)(10)^*(11 \cup 0))^*$
(c) $(01 \cup 10 \cup 0(10)^*00 \cup 00(10)^*0 \cup 1(01)^*11 \cup 11(01)^*1)^*$
(d) $(000111 \cup 001110 \cup 010101 \cup 011110 \cup 100011 \cup 101010 \cup 110001 \cup 111000)^*$

1 point

4. Which one of the following finite automata recognises the language described by the regular expression $(a \cup b)^*a(a \cup b)$:



1 point

5. Consider the context-free grammar $G = \{\ S \rightarrow aSbS \mid bSaS \mid \varepsilon\ \}$. With respect to the following claims, list in order whether each one of them is *true (T)* or *false (F)*. (Each correct answer is worth $\frac{1}{2}$ points.) *2 points*

(a) The string *ababb* is generated by the grammar G.
(b) Every string of the form a^nb^n, $n \geq 0$, is generated by G.
(c) The grammar G is ambiguous.
(d) The language generated by G is regular.

Total 10 points

PLEASE TURN OVER!

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Part II

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1. (a) Design a finite automaton that recognises the language

$$L = \{a^n \mid n \text{ is divisible by 2 or 3 (or both)}\}.$$

3p.

   (b) Give a regular expression describing the language L in part (a). *2p.*

2. (a) Design a context-free grammar that generates the language.

$$S = \{a^m b^n c^{m+n} \mid m, n \geq 0\}.$$

3p.

   (b) Prove that the language S in part (a) is not regular. *2p.*

3. Design a (deterministic, single-tape) Turing machine that recognises ("decides") the language S defined in the previous problem. (Present the Turing machine in terms of state or machine diagrams, rather than transition tables.) *5p.*

4. (a) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language, and explain their relation to issues in computer programming. *3p.*

   (b) Give an example of a language that is recursively enumerable, but not recursive. (You should provide a precise definition for the language, but need not prove any of its claimed properties.) Explain the significance of your example from the point of view of computer programming. *2p.*

Each problem 5 p., total 20 p.

Both parts combined total 30 p.

PLEASE TURN OVER!