T-79.148 **Spring 2005**
Introduction to Theoretical Computer Science
Tutorial 7, 8-11 March
Problems

Homework problems:

1. Design a right-linear grammar that generates the language

$$\{w \in \{a,b\}^* \mid \text{ contains the substring } ab \text{ exactly twice }\}$$

(Cf. Problem 3/1c.)

2. Consider the following grammar generating a certain type of list structures:

$$S \to (S) \mid S,S \mid a.$$

(a) Based on the above grammar, give a leftmost and rightmost derivation and a parse tree for the sentence "$(a,(a))$".

(b) Prove that the grammar is ambiguous.

(c) Design an unambiguous grammar generating the same language.

3. Consider the following context-free grammar:

$$\begin{aligned} S &\to TT \mid U \\ T &\to 0T \mid T0 \mid 1 \\ U &\to 0U00 \mid 1 \end{aligned}$$

(a) Describe the language of the grammar verbally.

(b) Show that the language is not regular.

Demonstration problems:

4. Prove that the class of context-free languages is closed under unions, concatenations, and the Kleene star operation, i.e. if the languages $L_1, L_2 \subseteq \Sigma^*$ are context-free, then so are the languages $L_1 \cup L_2$, L_1L_2 and L_1^*.

5. (a) Prove that the following context-free grammar is ambiguous:

$$\begin{aligned} S &\to \textbf{if } b \textbf{ then } S \\ S &\to \textbf{if } b \textbf{ then } S \textbf{ else } S \\ S &\to s. \end{aligned}$$

(b) Design an unambiguous grammar that is equivalent to the grammar in item (a), i.e. that generates the same language. (*Hint:* Introduce new nonterminals B and U that generate, respectively, only "balanced" and "unbalanced" **if-then-else-**sequences.)

6. Design a recursive-descent (top-down) parser for the grammar from Problem 6/6.